R 370

Technical Report

A GRAPHICAL PERT ANALOG

April 1965

U. S. NAVAL CIVIL ENGINEERING LABORATORY

Port Hueneme, California

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INTRODUCTION

The Graphical PERT* Analog (GPA) is an integrated non-computer method of planning and scheduling the component activities of a project in terms of both time and cost. Essentially, GPA is a graphical approach to the PERT and CPM** concepts of project analysis. Thus anyone who is familiar with PERT or CPM techniques will be able to use GPA readily for construction and maintenance tasks of limited complexity for which planning and scheduling are desirable, but for which it is not practical to use a digital computer. As is generally true of PERT/CPM planning and scheduling methods, GPA can assist management to utilize available productive resources effectively; however, compared to other non-computer methods presently used, GPA techniques are more straightforward and require minimal arithmetic calculations.

GPA comprises three topics which will be discussed in separate sections: (I) the analysis of a project's component activities and a synthesis of these activities into a time-phased flow diagram schedule; (II) the derivation of least-cost schedules for accelerated projects; and (III) the determination of predicted cumulative direct costs as a function of time for a given project duration. Practical applications of the method will be explained and exemplified.

SECTION I. DEVELOPMENT OF THE FLOW DIAGRAM SCHEDULE

The GPA flow diagram schedule (Figure 1) has the form of a bar graph which depicts a forecast of the project's progress in time and also shows the interdependence of the component activities involved. An important advantage of this type of flow diagram over the usual PERT flow diagram and the Gantt chart is that activity pursuit durations, schedule-latitude time intervals, and the critical path are intrinsic to it. Thus the GPA flow diagram schedule alone provides a convenient means of positive control for evaluating and directing the progress of a project. The graphical approach of GPA facilitates planning because it imparts considerable insight during development of the flow diagram and eliminates the need for arithmetical calculations to determine the critical path and schedule latitude. In addition the GPA flow diagram serves as the basis for effective methods of project cost analysis, as will be seen in Sections II and III.

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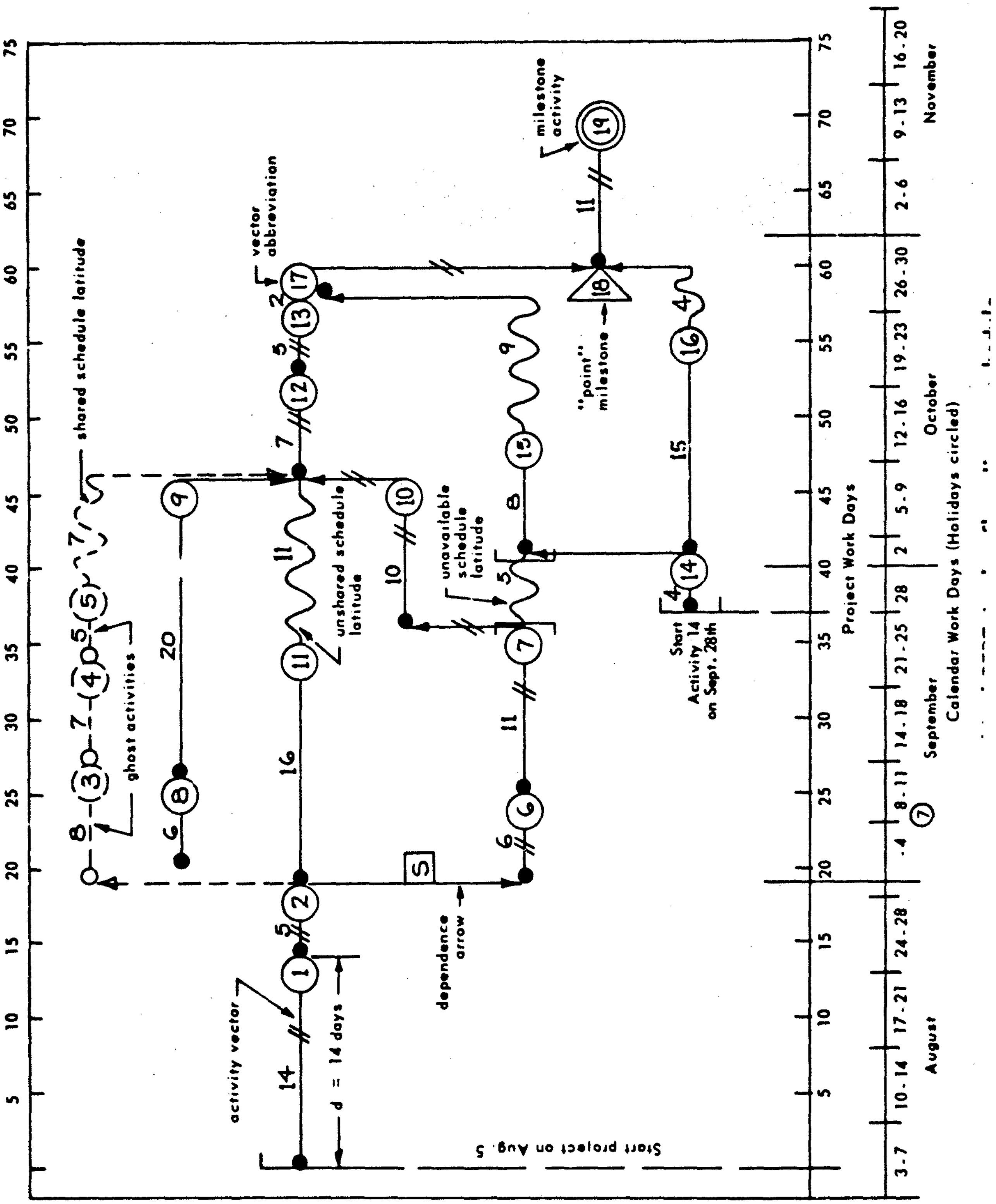
* Program Evaluation and Review Technique. "PERT" is used in this report as a generic term for all project analysis techniques involving the concept of a critical path.

** Critical Path Method.


activity vector
d = 14 days
dependence arrow
Start project on Aug. 5
ghost activities
shared schedule latitude
unshared schedule latitude
unavailable schedule latitude
vector abbreviation
"point" milestone
milestone activity
Start Activity 14 on Sept. 28th
Project Work Days
Calendar Work Days (Holidays circled)
August
September
October
November

THEORY

The basic graphical concept behind GPA is that activities, or operational components, are interpreted as elements, called "activity vectors," which have a starting point, a pursuit duration, and a completion point in time. These activity vectors are arranged along a time axis according to the logic sequence stemming from their interrelationships. Finally, the logic sequence interrelating the activities is depicted to complete the flow diagram schedule and reveal its salient characteristics. Two basic principles govern the construction of the flow diagram:

1. The pursuit of an activity can only be shown along a horizontal time axis.
2. The dependence of one activity upon another can only be shown along a vertical axis.

GPA Graphic Symbols

The next step in becoming acquainted with GPA is the identification of its graphic symbolism, or vocabulary.

Indicates the start event of an activity.

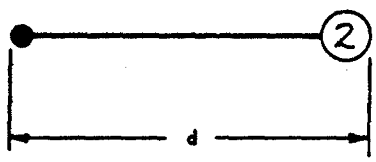
Indicates the pursuit of an activity, and can only be drawn horizontally toward the right.

Indicates the completion event of an activity. An inscribed number or letter identifies the activity.

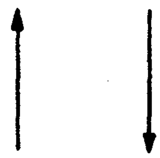
milestone as part of an activity vector

"point" milestone

Indicates the attainment of a milestone — or that a substantial portion of work has been completed. Milestones may stand on the flow diagram as a point in time without being part of an activity vector — for example, as an important decision. In this case, the symbol becomes a triangle as shown.



Illustrates how the event and pursuit symbols above are combined to form an activity vector. The dimension (d) is a convenient scale length which corresponds to the duration of the activity (e.g., 1/8 inch = 1 day).



Indicates the dependence of one activity upon another, and can only be drawn vertically.

Indicates the critical path, which is the longest continuous path through the project. Activities on the critical path must be started and completed on time to maintain the project on schedule.

Indicates schedule latitude — time intervals when effort need not be expended on an activity to complete the project on schedule. Schedule latitude is similar to PERT "slack" and CPM "float." In certain cases, schedule latitude may be shared between two or more activities. Occasionally, constraints to the schedule may result in latitude being shown but actually not available.

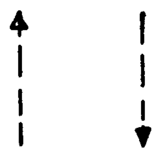
Ghost Activities

The following set of symbols applies to contingent activities which may be necessary (for example, redesign of a component), or to an alternate activity sequence. These activities are called "ghosts."

Indicates the start of a ghost activity.

Indicates the pursuit of a ghost activity.

Indicates the completion of a ghost activity.

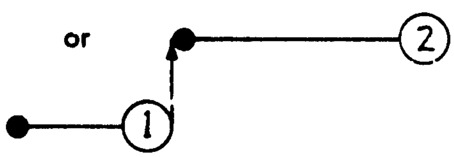


Indicates the dependence of activities, one or both of which are ghosts.

Indicates schedule latitude with respect to ghost activities.

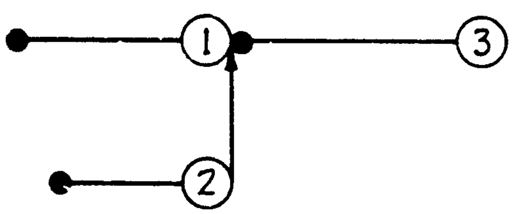
Inferences From Combinations of Symbols

or

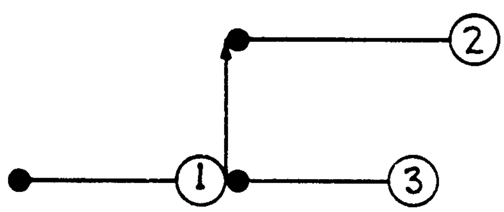


Activity 1 must be completed before activity 2 can be started.

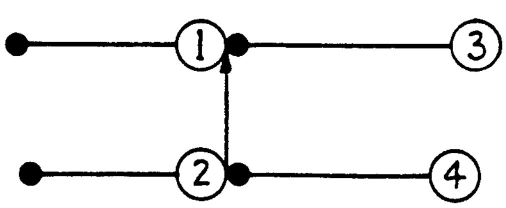
(Carefully note these equivalent forms.)



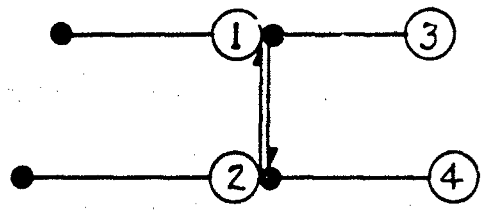
Both activities 1 and 2 must be completed before "3" can be started.



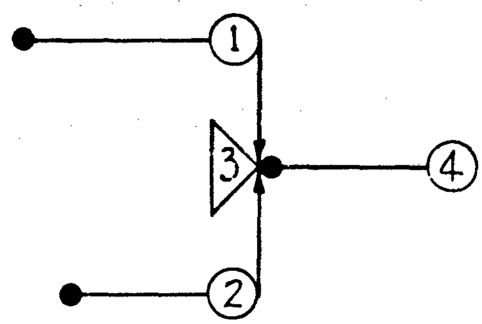
Activity 1 must be completed before "2" or "3" can be started.



"4" may be started as soon as "2" is completed. However, both "1" and "2" must be completed before "3" can be started.

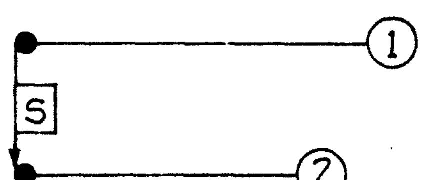


Neither "3" nor "4" can be started until both "1" and "2" are completed.



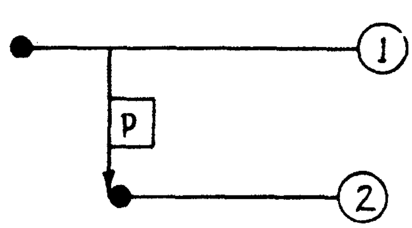
"3" is a point milestone which depends on the completion of "1" and "2" and governs the start of "4." ("3" involves no directly related activity in this case, but may in other cases.)

There are 6 days (or other time unit) of shared schedule latitude between the completion of activity 1 and the start of activity 2. "1" may be completed 6 days late and thereby stretched to a duration of 11 days, or "2" may be started 6 days early. In any event, the total stretching of "1" and "2" cannot exceed 6 days. In certain cases, the entire activity vector can be transferred forward or backward along the latitude notation. The concepts of available and unavailable latitude will be discussed later.



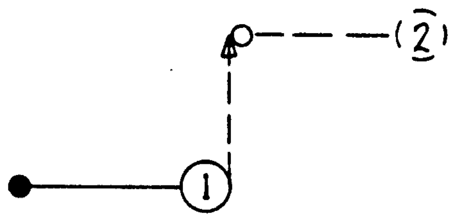
Activity 1 must be started before "2" can be started.

(Complex concurrency)



Some percentage (usually expressed in time units) of activity 1 must be completed before "2" can be started.

(Complex concurrency)



"2" might be necessary upon the completion of "1."

PRACTICAL APPLICATION

Construction of the Flow Diagram

In brief, the recommended procedure for constructing a GPA flow diagram is to list the activities to be considered, estimate a duration time for each activity, and determine the sequence of operations. The activity vectors and milestones are drawn on another sheet of paper and cut into separate pieces. These pieces are placed on a diagram blank according to the sequence of operations, and the dependence arrows and other symbols are drawn on the blank to delineate the sequence. The step-by-step procedure is as follows:*

1. Break down the project into important activities and major milestones, and list them by short descriptive phrases and identifying letters or numbers on a sheet of paper, which will be referred to hereafter as the worksheet. (See Table I for the worksheet which governs the flow diagram shown in Figure 1.) The degree of detail of the breakdown will depend on the amount of control required by management. Activities must be distinct and should pertain directly to the fulfillment of the project. An activity usually represents an element of work, a process, a procurement, or a waiting time and should be considered if it satisfies <u>any one</u> of the following conditions:

   a. Its completion represents a determinate goal or event, or

   b. The start of subsequent activity depends on the completion of the activity in question, or

   c. Completion of the activity represents a time when effort is passed from one individual or group to another.

There is no need to list the activities in any particular order because the logic sequence is determined in the next step; however, the worksheet will be easier to develop and use if the activities are listed approximately according to the operational sequence of the project.

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* The basic technique presented in Section I for developing the GPA flow diagram schedule has also been presented in Reference 1.

2. For each activity, list the activity or activities which must be completed just prior to beginning the activity in question. Since the operational logic sequence of the project (and flow diagram) is determined in this step, careful thought and judgment should be exercised.

3. Now, for each activity listed, note an estimate of the number of time units (hours, days, weeks, etc.) required to complete it. The time unit chosen must be used consistently throughout the analysis. As with other PERT-type methods of project analysis, GPA uses the "crew-day" estimating approach rather than the "man-day" approach. Note that GPA uses a single-duration time estimate for each activity. The estimate is based on prior experience with work which is similar to, or related to, the activity in question. Sometimes the estimate must simply be an "educated guess." In either case, the estimate should be made carefully.

It must be recognized that individual estimators will tend to bias their duration estimates, and consequently bias the project schedule. To reduce bias, the estimator should consider each activity as a separate entity and not let estimates affect one another. Contingency should be used sparingly, because contingency will pyramid when the schedule is developed; however, work tie-ups which can be anticipated for a given activity should be considered. When there are several factors which may affect the timely performance of an activity, it is sometimes helpful to utilize a formula which was developed for the PERT system:[2]

$$\text{duration (d)} = \frac{O + 4E + P}{6}$$

where O is an optimistic time estimate
E is the expected time estimate
P is a pessimistic time estimate

By using this formula, the estimator can account for the divergence between optimistic, expected, and pessimistic time estimates and derive a probable single-duration time estimate. However, this formula has not been rigorously proven to be statistically valid; therefore, it is recommended only as a tool to help the estimator arrive at judicious time estimates.

Milestones need not be associated with an activity duration because they may stand as a point in time to represent important decisions or the completion of an especially significant portion of work. If the start of completion of a given activity or milestone must occur by a certain date which is beyond the control of the planner, the activity event or milestone is critical, and the date in question should be noted.

Table I. Basic Graphical PERT Analog Worksheet

Description	Department Code	Activity Code	Follows	Duration (days)
		1	Start 5 Aug	14
		2	1	5
		3_g 1/	2	8
		4_g	3_g	7
		5_g	4_g	5
		6	11(s) 2/	6
		7	6	11
		8	—	6
		9	8	20
		10	7	10
		11	2	16
		12	5_g,9,10,11	7
		13	12	5
		14	Start 28 Sept	4
		15	7,14	8
		16	14	15
		17	13,15	2
		18_m 3/	16,17	—
		19_m	18_m	11

1/ "g" stands for ghost.

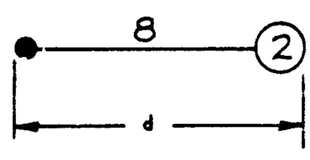
2/ (s) stands for complex concurrency (activity 6 cannot be started until activity 11 has been started — see step 6 of "Construction of the Flow Diagram").

3/ "m" stands for milestone.

Having performed these three steps, the worksheet for the basic flow diagram schedule is complete. The "Department Code" column of the sample worksheet shown in Table I provides a space, if desirable, for listing the individuals or groups who actually perform the activities.

4. Make up a GPA flow diagram blank which has the total expected or allotted task time divided into days, weeks, or months. Only working days should be considered. The selected time intervals are drawn to a convenient scale — for example, 1/8 inch = 1 day. Once the working-day time unit intervals have been set up, calendar days can be associated with them by noting which calendar date corresponds with the first day on the diagram blank and then continuing with the corresponding days and dates, carefully avoiding nonworkdays such as weekends and holidays (see Figure 1).

5. On a separate sheet of heavy paper, draw the series of activity vectors and milestone triangles for the activities and point milestones already determined. Remember that the length of the activity vector, (d), corresponds to the number of days (or other time units) needed to complete the activity in question, and that the scale used must be the same as that used to make up the flow diagram blank. Draw the numerical value of "d" for each activity just above the activity pursuit line and identify the activity in the event circle, thus:



d = 8 days to complete activity 2

Vectors for activities having very short durations sometimes must be abbreviated by omission of the pursuit line for proper representation on small-scale GPA flow diagrams. These abbreviations are shown in Figure 2. Milestones may or may not be a part of an activity vector, depending upon the nature of the milestones.

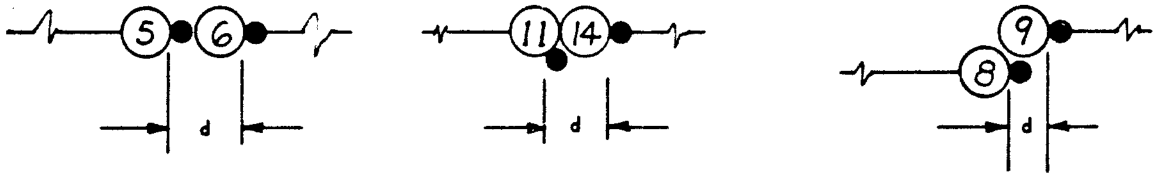
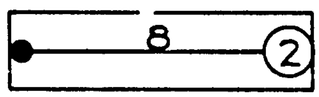


Figure 2. Typical activity vector abbreviations.

Now cut out the activity vectors and milestone triangles into separate pieces so that they look as follows:



6. Lay, but do not affix, the assorted activity vector pieces on the flow diagram blank and lightly sketch in dependence arrows so that meaningful relationships in time between the activities begin to take place according to their logical sequence and GPA language. The planner might start with the first activity in a logical series of activities and build the flow diagram forward in time. If the completion date of the final activity has been set, it is also feasible to work backward in time by using the final event as a reference. Critical activity events must occur by a certain date; thus they provide convenient references for building the flow diagram forward or backward in time.

If the complexity of the project is such that this method of initial layout of the diagram seems cumbersome, a rough-draft layout often helps the planner to visualize and simplify the format of the flow diagram schedule. In making such a rough draft, it is recommended that the notation suggested by J. W. Fondahl, of the Construction Institute, Stanford University, be used.[3] Activities are represented by circles with inscribed identifying letters or numbers — the same as the GPA "activity completion" circles. Similarly, milestones are represented by double circles if they are part of an activity and by triangles if they represent points in time. Activity and milestone interdependence is shown with arrows. The primary object of the rough draft is to portray the logic sequence of the project; thus the activities and "point" milestones are connected without regard to time, except that the diagram should "flow" from left to right, as shown in Figure 3.

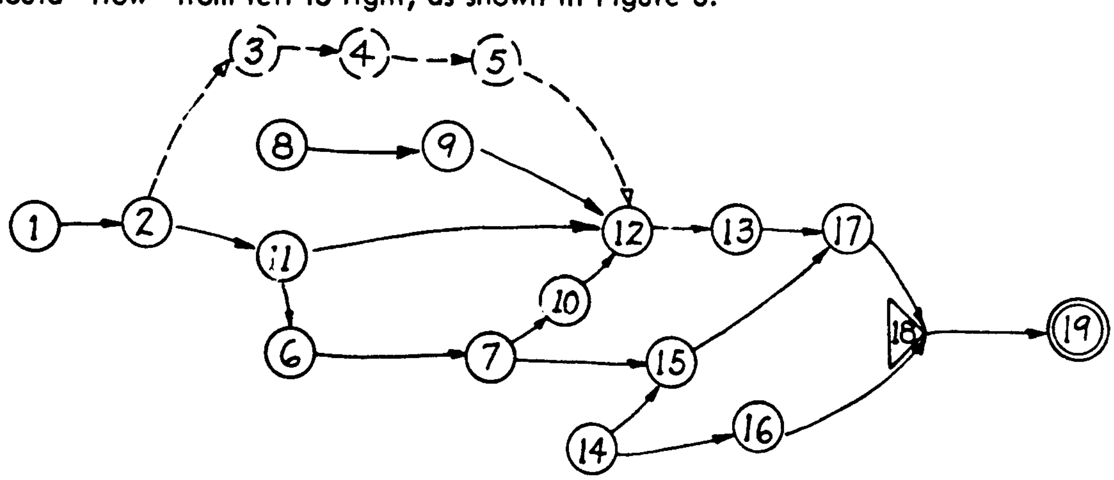


Figure 3. Rough draft of the GPA flow diagram.

As the relationships between activities on the GPA flow diagram become more clear, due regard should be given to activities which can occur simultaneously. Planning simultaneous activities within the limitations of available productive resources can lead to high levels of efficient progress. Exceeding these limits can cause production costs to soar. Thus, the planner must be careful not to overextend the available productive resources unless emphasis is to be placed on an accelerated or crash program effort. Occasionally manpower and/or equipment limitations will require that an activity follow another activity even though they have no intrinsic relationship which can be indicated on the worksheet. This constraint should be delineated by a dependence arrow which is appropriately labeled. The planner can also consider alternate activity sequences acceptable to the program by simple manipulation of the activity vectors, or he can vary activity durations by substituting alternate vector lengths. In certain cases where one activity depends on another it is possible to begin the dependent activity when the governing activity has been only partially completed. The laying of pipe in a long trench is an example — it is not necessary to dig the entire trench before beginning to lay pipe. This situation is called "complex concurrency," and it has been depicted in the group of inferences illustrated in the explanation of the GPA symbols (see "Theory"). Sometimes a given percentage of the governing activity must be completed before the dependent activity can be undertaken; thus one should be careful in determining at what point the dependent activity can be started in relation to the progress of the governing activity (see Figure 4 below). When considering complex concurrency it is necessary to review the duration of the dependent activity in relation to that of the governing activity; in most cases the dependent activity cannot be finished before the governing activity has been completed. Additionally, because of production resource limitations, it may be necessary to extend the duration of the governing activity. Remember that when duration times were assigned in Step 3, activities were considered as separate entities.

It will probably become apparent at this point that some of the activity vectors do not correspond properly in time so that interdependence can be delineated by vertical lines as prescribed earlier. Consider the sequence: "6" must be completed before "8" can be started. Because of logic or production limitations, the activity vectors might be located on the flow diagram as shown on Figure 4.

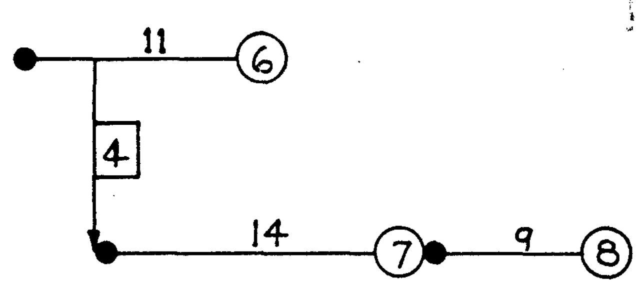


Figure 4. An example of complex concurrency.

In this case, schedule latitude is used to complete the logic sequence. The duration of schedule latitude can be directly determined by scale measurement, and should be recorded above the latitude symbol as shown in Figure 5.

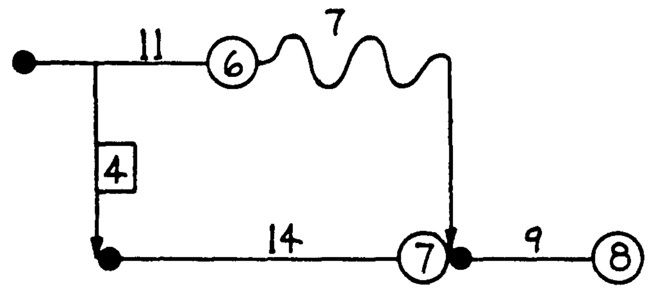


Figure 5. The logic sequence completed by schedule latitude.

Ghost activities are accounted for in this planning step by allowing for sufficient schedule latitude in the pertinent portions of the flow diagram. In an accelerated or crash program, there may be instances when allotment of latitude for ghost activities is not possible because of time limitations. For this case the ghost activities are not "programmed" into the flow diagram — they only stand to indicate potential problem areas and/or conceivable alternate activity sequences (see Figures 6 and 7).

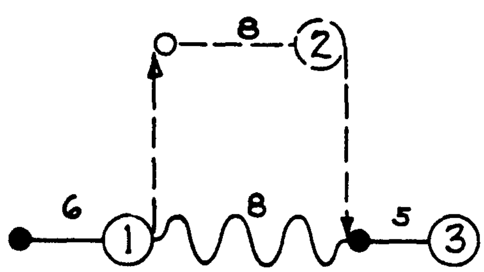


Figure 6. A ghost activity programmed into the flow diagram.

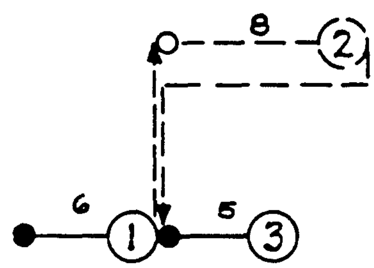


Figure 7. The ghost activity not programmed — it stands as a potential problem.

7. Once the activities appear to be located on the flow diagram blank so that they satisfy logic requirements and production cost limitations within the time framework, the activity vector pieces may be cemented in place. Rubber cement does nicely because it is easy to work with, and the pieces can be pulled off in the event a revision of the flow diagram is necessary. Now the schedule latitude, activity-dependence arrows, and other symbols can be drawn in boldly to complete the flow diagram as in Figure 1. One can see that the GPA method has yielded a meaningful schedule of the project. Besides noting the general features of the flow diagram schedule, the reader should be aware that the significance of activity durations, the critical path, and schedule-latitude intervals is apparent by inspection. Thus the effect of any change to the program can be traced at a glance.

The reader can gain actual experience in laying out a GPA flow diagram by turning to Appendix A where a worksheet, a flow diagram blank, and a set of activity vectors have been provided for convenience. The project presented in Appendix A will be discussed in Sections II and III.

Interpretation of the Flow Diagram Schedule

Turn to Figure 1. The critical path, which is the longest continuous path through the project is 1-2-11(s)-6-7-10-12-13-17-18_m-19_m; these activities must be performed on schedule if the project is to be completed on time. Note that the start of activity 11 is on the critical path. Although the completion of activity 11 does not appear to be critical, this activity must be performed so that it does not interfere with the performance of activity 6, which is critical. Ghost activities 3, 4, and 5 are contingent upon the outcome of activity 2. If these activities must be performed, the presence of a 7-day schedule latitude in the ghost path indicates scheduling flexibility. For example, the latitude could be apportioned among the three activities, or the performance of the entire ghost sequence could be delayed by 7 days without interfering with the critical path. This kind of schedule latitude is called "shared schedule latitude." A more complex case of shared schedule latitude is found between activities 15 and 17. This latitude can be apportioned among activities 14 and 15; however, there is one limitation: the completion of activity 14 cannot be delayed by more than 4 days without delaying the completion of activity 16 to the point where it interferes with the start of activity 19_m, which is critical. Consequently, only 4 days of the 9 days of schedule latitude after activity 15 are available to activity 14 and shared with activity 15. Thus it follows that (9 - 4) = 5 days of this latitude are available to activity 15 only. Schedule latitude which is available to only one activity is called "unshared schedule latitude.' Another example of unshared schedule latitude is found after activity 11. Of course this latitude is only available to the completion of activity 11 since the start of "11" is critical. Schedule latitude which cannot be taken by an activity is called "unavailable" to that activity. It should be noted that a given amount of schedule

latitude can be available to one activity and unavailable to another, as, for example, in the previous cases of unshared latitude. An example of totally unavailable schedule latitude can be seen between activities 7 and 15. Activity 7 is critical and the start of activity 15 is constrained by activity 14, which has a fixed starting date. In this situation, the schedule latitude serves merely as a link between these activities to indicate an operational interrelationship.

It has been argued that the three types of schedule latitude — unshared, shared, and unavailable — should have special symbols; however, the answer to this argument is that the system of symbols would unnecessarily complicate the GPA flow diagram because the nature and implications of schedule latitude can be easily traced. Nevertheless, the use of a special symbol for unavailable schedule latitude may be desirable to prevent a serious misinterpretation of the flow diagram. For this purpose a bracket is suggested to close off activities from unavailable schedule latitude (see Figure 8). In this particular case the 10 days of latitude between activities 1 and 4 is not completely available because a delay in activity 1 by more than 4 days will ultimately force activity 2 to interfere with the start of activity 5, which is critical. Path 1-2-5 is termed "restrictive" in this case. The bracket indicates that, of the 10 days of schedule latitude, only 4 days are available to activity 1.

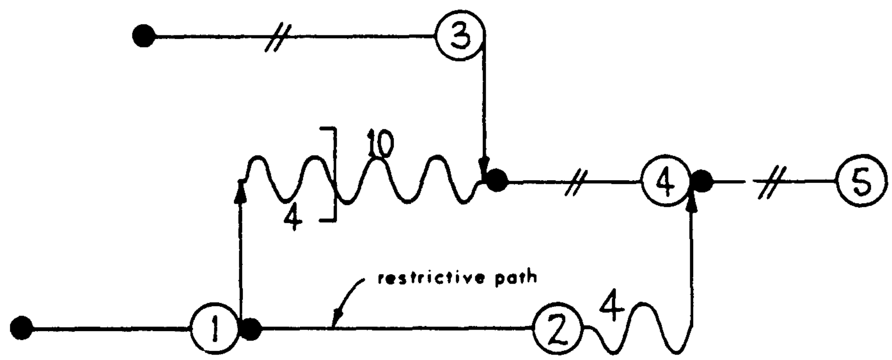


Figure 8. Use of the bracket symbol to delineate between available and unavailable schedule latitude.

Before leaving the subject of flow diagram interpretation, the matter of activity events should be discussed. GPA is an activity-oriented system, whereas PERT is usually considered an event-oriented system. This minor difference rests primarily on whether one places emphasis on activities as integral elements or upon the "start" and "completion" of activity which leads to a definable goal. Even though activities are manipulated as integral elements in the GPA system, the start and completion events are depicted symbolically and have significance. For example, with GPA, one can make a distinction between activities having one critical event (start or completion) and critical activities which comprise critical start <u>and</u> completion times. But a more general aspect of activity event times involves the concepts of

"earliest event times" (EET) and "latest event times" (LET). When activities are shown at their earliest event times, they can be scheduled no earlier, and the converse is true for activities scheduled at their latest event times. The possibility of EET and LET with respect to an activity implies a degree of scheduling flexibility (presence of schedule latitude); consequently, EET and LET have no significance with respect to critical events and activities — they are, in fact, synonomous. An example of activities shown at their earliest event times is found in Figure 1, with ghost activities 3, 4, 5. Note that the 7 days of schedule latitude are programmed after the activities. To show these activities at their latest event times, one merely transposes the activity sequence and schedule latitude. An important result of this action is that these ghost activities, if performed at LET, would be critical. The activity sequence 8 - 9 is shown at LET and is critical. In this case schedule latitude between activity 8 and the start of the project (5 Aug) is implied. Unless there were productive resource limitations, there is no reason why these activities could not be moved back to allow for schedule latitude between activities 9 and 12 and thereby make the 8 - 9 activity sequence uncritical.

Project Review

A carefully thought-out GPA flow diagram affords a convenient check on the progress of a task as it unfolds. A continuous record of progress is maintained by drawing a colored line parallel to an activity pursuit line to indicate the percentage of work accomplished on a particular activity. In this way the effect of unforeseen problems on the task work loads can be immediately noted to facilitate the redirection of needed effort, if possible.

As will be seen in Section III, the GPA flow diagram can be used for cost-control purposes by plotting estimated cost as a function of time with respect to the various activities directly on the diagram. Actual costs are then plotted as work proceeds to help determine the financial status of the project and forecast future cost problem areas.

A Summary of Helpful Hints

Practical application of GPA techniques over the past 2-1/2 years has pointed up a number of hints which can assist the planner and management to realize the full potential of this system.

1. Plan to spend most of your planning and scheduling effort on developing the GPA worksheet. As in other systems of project analysis (PERT and CPM), the worksheet determines the nature of the flow diagram, so a few minutes of extra care on the worksheet can save time in the long run.

2. Remember that activities are valid if they represent either an element of work, a process, a procurement, or an enforced waiting period. For example, the waiting period required by a procurement contract should be considered as an activity even though some other organization actually performs the work.

3. When making duration time estimates, consult with those individuals who will be responsible for activity performance unless there is precedence from prior work or time standards have been established. Consultation can help to reduce bias in estimates and promote mutual cooperation. Unfortunately, this approach is not always possible.

4. It is generally a good idea to label each activity according to the individual or group who will perform the work. A column can be added to the GPA worksheet for this purpose (the "Department Code" column in Table I), and appropriate identifications can be written directly on the activity vector strips.

5. Once the GPA worksheet has been completed, arrange to have it checked by someone else who can understand the project. It is surprisingly easy to overlook necessary activities and activity interrelationships.

6. Experiment with the layout of the flow diagram. You will find that it is possible to arrange a valid flow diagram in a number of ways because the dependence between activities can be shown by arrows, or simply by placing the activity vectors end to end. So that the final diagram will be easy to read, work toward minimizing the number of dependence arrows, and, whenever possible, avoid vector placements which require that arrows cross over other activity vectors. A rough draft of the flow diagram is often very helpful in simplifying the final layout (see Figure 3).

7. During construction of the flow diagram, additional activity relationships often become apparent — even when the worksheet has been planned carefully. Therefore, do not lay out the diagram dogmatically, but be flexible in your thinking. (Remember that during the planning phase the activity vectors are merely laid on the diagram blank so that they can be moved easily.)

8. Keep in mind that there are several basic types of constraint:

   a. The start of activity___depends on the completion of activity___.

   b. The start of activity___depends on the start of activity___.

   c. ___% (in time units) of activity___must be completed before activity___can be started.

Also consider why a constraint is imposed. — Is it the intrinsic nature of an activity which governs the performance of another activity? Or are the constraints imposed by manpower and/or equipment limitations? When constraints are the result of limited production resources, the dependence arrows should be labeled appropriately.

9. When the flow diagram schedule has been completed, the critical path should be determined and identified. Once the critical path has been noted, all schedule latitude appearing on the diagram should be inspected to determine whether it is unshared, shared, or unavailable. Unavailable schedule latitude occurs when latitude shown on a particular path cannot be taken by an activity because of restrictive activity interaction with another path. Unavailable schedule latitude with respect to a given activity cannot be used without adversely affecting the timely performance of activities on the critical path; thus cases of unavailable schedule latitude should be conspicuously identified (see Figure 8).

10. Make sure that all individuals who are responsible for activities appearing on the flow diagram are shown how they contribute to the overall effort. Experience has shown that when individuals are informed as to why they must adhere to a given schedule, they will be much more cooperative. Distribution of the diagram to the field often requires that it be reproduced. A method of flow diagram construction which permits reproduction by standard dry reproduction techniques is given in Appendix B.

11. Remember that the GPA schedule is a time-phased program of what you expect to accomplish; ideally it is the best workable schedule from the planner's viewpoint. However, the schedule is subject to exigencies which may arise in the field. Thus, schedules should be reviewed periodically and up-dated as necessary.

A Graphical PERT Analog Device

The practical application of GPA techniques can be greatly facilitated by the use of a device such as the PERT Analog illustrated in Figures 9 through 13. Designed and constructed at the U. S. Naval Civil Engineering Laboratory, this device has proved to be very successful because of its convenience and flexibility.

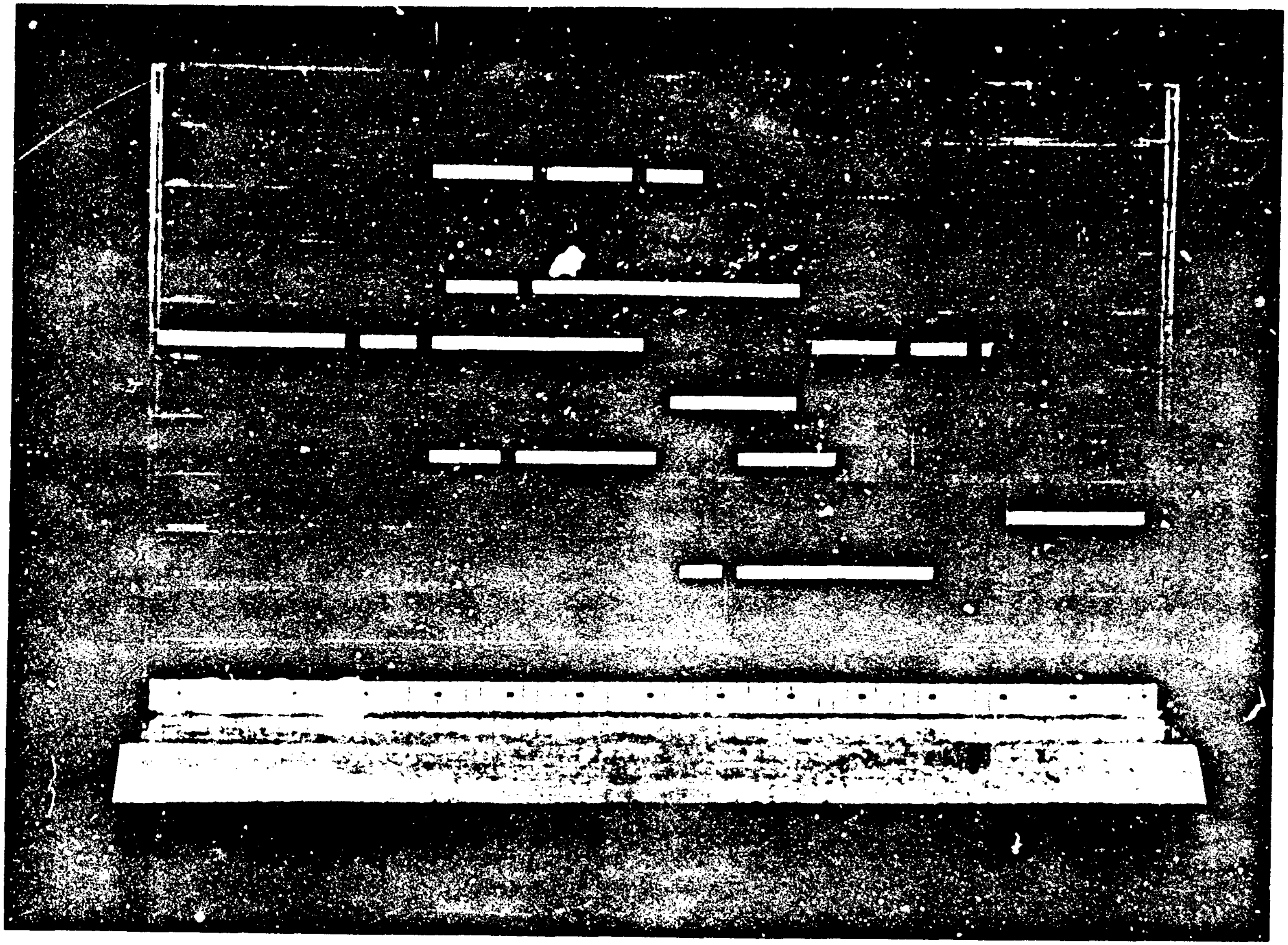


Figure 9. Overall view of the PERT Analog, which has been programmed for the project presented in Section I.

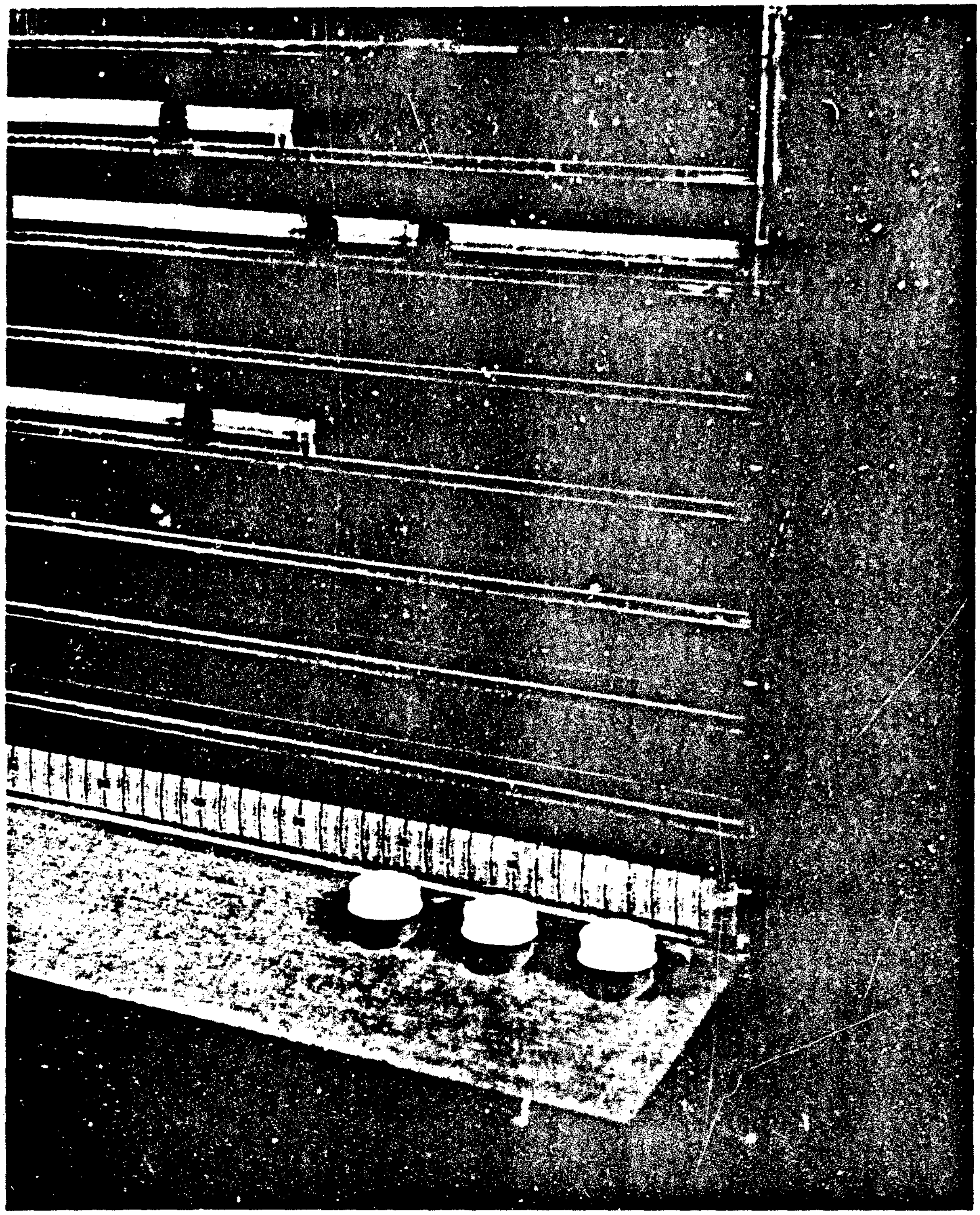


Figure 10. Racks are provided on the reverse side of a plexiglass backboard panel to hold activity vectors. The activity vectors can be moved from rack to rack and slid freely within each rack.

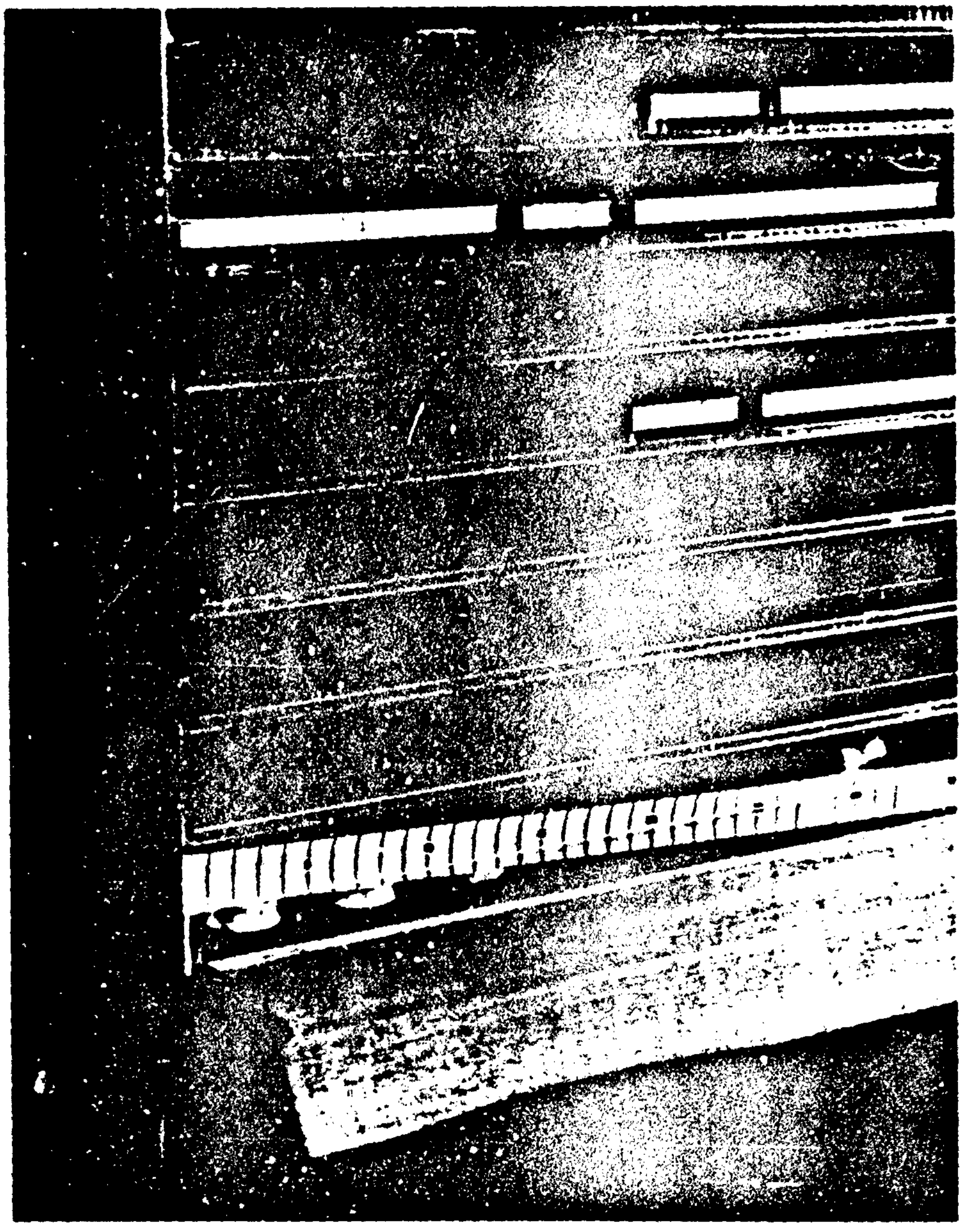


Figure 11. A time scale is inserted behind a plexiglass coverplate. Alternate time scales are stored in the three film cans seen in Figure 10. Vertical lines inscribed on the plexiglass backboard panel correspond to divisions on the time scale; these lines aid in the alignment of the activity vectors to the time scale. Activity interrelationships, schedule latitude, and other notations are drawn with a grease pencil directly on the obverse side of the backboard panel.

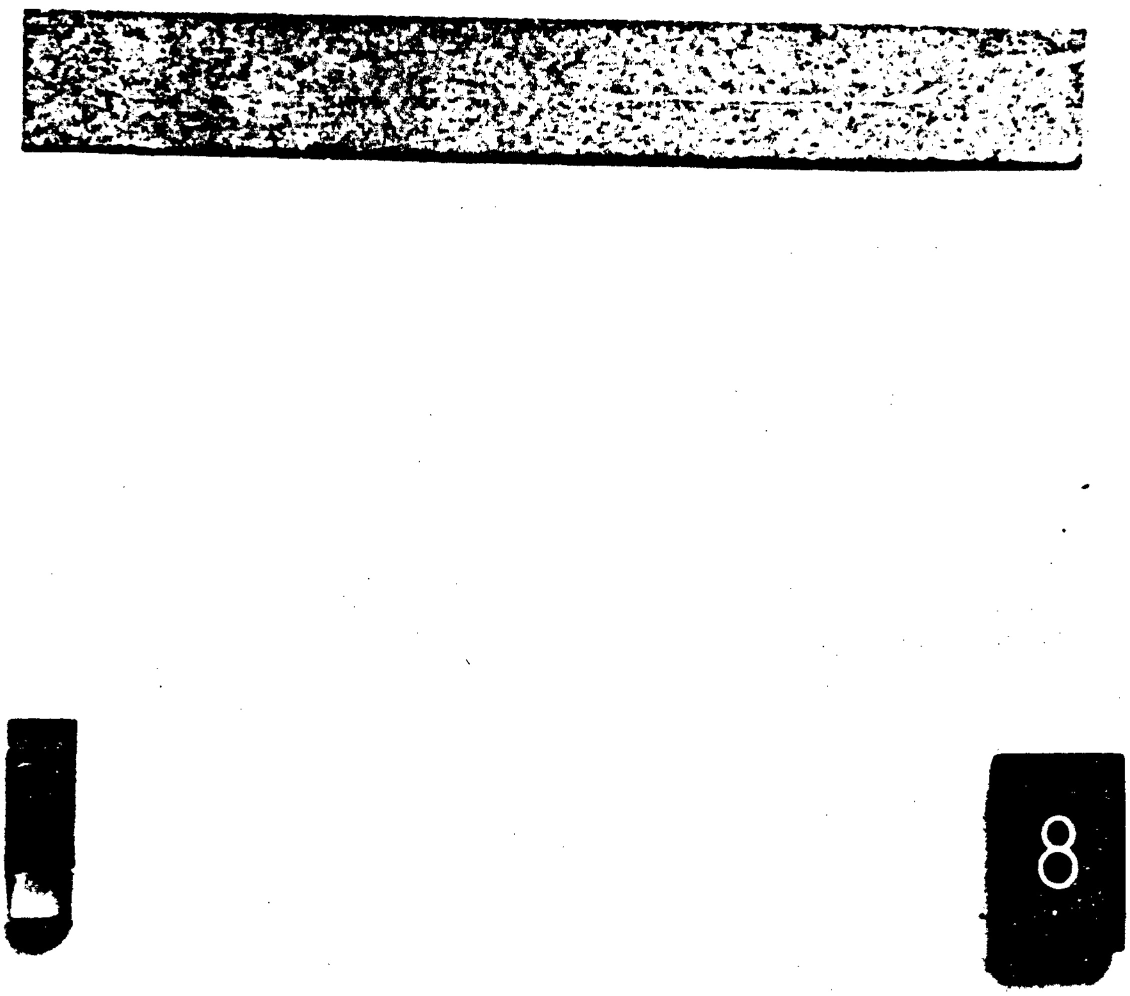


Figure 12. The activity vectors consist of a plastic "start" clip, a cardboard "pursuit" strip cut to scale length, and a plastic "completion" clip into which the activity identification numbers have been inscribed. Activity vectors are made up quickly by first cutting cardboard strip stock to length as determined by the activity duration and the time scale used, then the "start" and "completion" clips are slipped over the ends of the cardboard strip.

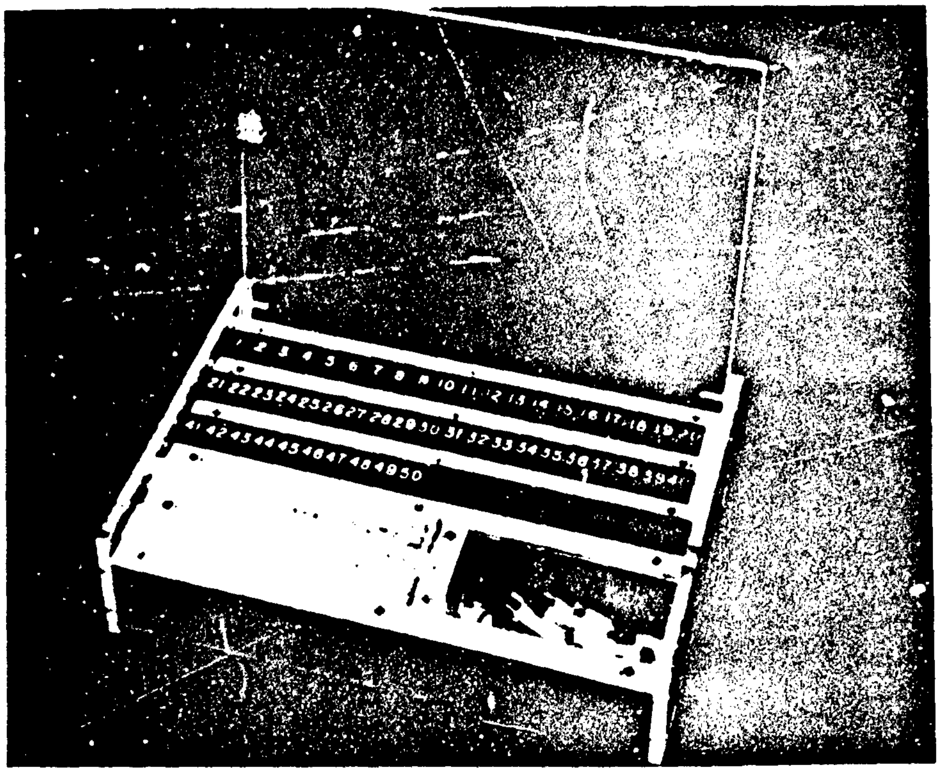


Figure 13. The activity vector "start" and "completion" clips are conveniently stored in this case.

CONCLUDING REMARKS TO SECTION I

The GPA technique provides a relatively simple means of systematic project planning and scheduling because the graphical method is straightforward and arithmetic calculations are minimized. It may seem that considerable time and judgment are required to develop a GPA flow diagram. Actually, GPA takes much less total time than other non-computer planning methods to achieve a meaningful and easily understood schedule of the project. Furthermore, the time to construct a GPA flow diagram can be reduced through the use of special devices such as the PERT Analog. The need for judgment cannot be avoided, but the graphical approach of GPA promotes sound judgment in making planning decisions.

The primary emphasis of Section I has been directed toward the analysis of activity interactions with respect to time, with secondary consideration given to cost. Although the material contained in this section can be effectively used alone to analyze a project, special cost problems arise when accelerating a project, and also when it is desired to trace the progress of a project relative to cost. These topics follow in Sections II and III respectively.

SECTION II. PROJECT ACCELERATION AT LEAST COST

It has been shown in Section I that the GPA flow diagram schedule quickly reveals pertinent information necessary to the understanding of a project's operational aspects. Scheduled activity start and completion dates, the critical path, the total project duration, and unshared, shared, and unavailable schedule latitude associated with noncritical activities can be determined by inspection. Moreover the effect of changes to the schedule can be seen readily. It is because the GPA flow diagram is particularly well-suited for rapid information retrieval that the following practical method of least-cost scheduling for accelerated projects is possible.

BACKGROUND

The flow diagram schedule shown in Figure 14 gives some insight into how a project might be accelerated. Note that this project is not the one discussed in Section I. The critical path (3-13-15-17-18) alone determines the project's duration. One or more of these activities on the critical path must be compressed to shorten the duration; the compression of any other activities at this time would be useless. But consider the effect on the schedule of compressing activity 15 by 2 days. Examination of the flow diagram will reveal that a second critical path (1-5-7-11-17-18) is created because the completion date of activity 15 has been moved up to the point where it corresponds with the completion of activity 11. Further compression of activity 15 by one more day requires the compression of an activity on the new critical path and also leads to a third critical path (2-10-14-16-18) because the completion of activity 17 now corresponds with the completion of activity 16. The point of this discussion is that acceleration of a project can change the character of its schedule significantly. The need to decide which activities should be compressed, and by how much, creates the complex problem of project acceleration at least cost. The following step-by-step procedure which can help to solve this problem is not unique in its fundamental concepts, just as the Graphical PERT Analog itself is not unique; however the implementation of these basic concepts, which rests on the GPA flow diagram, is a fresh approach to least-cost scheduling methods.

Briefly the method is as follows: The activity costs associated with normal and crash duration times are determined. This data is used to find activity-compression cost factors (daily cost to compress). The activity duration ranges (crash - normal) and compression cost factors are compiled on a cost-control diagram which is constructed during the compression process, and which follows the format of the project's GPA flow diagram. Project acceleration is begun by compressing, as necessary, the critical path, found from the flow diagram. Activity-compression cost factors are utilized to accomplish compression at least cost. Paths are added to the cost-control diagram in order of decreasing criticality (found from the flow diagram) until a path

can be added without requiring activity compression. For each path added, compression is achieved by first absorbing schedule latitude, and then compressing activities with regard to network restrictions and compression cost factors. Overall project acceleration at least cost is assured by cost investigations which simultaneously consider the cost interactions of all paths on the cost-control diagram.

PROCEDURE

Intermediate Project Acceleration

As recalled from Section I, the first five columns of the project worksheet (Table II), from which the flow diagram schedule of Figure 14 was developed, are concerned with the operational aspects of the project. Column 1 is merely a short description of each activity, column 2 indicates who actually does the work, column 3 gives the activity an identification number, column 4 indicates what activities must be completed just prior to starting the activity in question (the sequence of operations), and column 5 shows the normal time duration for each activity. For the purposes of cost scheduling, the definition of normal time duration is important. Normal time is considered to be that period (in crew-hours, crew-days, etc.) during which the activity can be performed at lowest cost. The determination of a least-cost time duration for each activity is a matter of judgment based on experience and any guidelines which the planner-and-estimator may have at his disposal. A detailed discussion of minimizing activity costs is beyond the scope of this report, but it can be said generally that one of the objectives of the planning process has long been the reduction of costs to a practical minimum. Thus the planner-and-estimator is not confronted with anything particularly new in this respect. Returning to the project worksheet, consider the next two columns: crash duration and crash cost. A crash duration is considered to be the practical minimum amount of time it would take to perform a given activity if additional manpower, more equipment (capacity), and cost premiums for expediting material procurements, etc., were authorized. Here again, the planner-and-estimator must exercise judgment based on experience and whatever guidelines he has to arrive at reasonable figures.

Although a quantitative definition of normal and crash durations for all situations is not possible, one can certainly conclude that the compression of an activity from its normal duration usually results in a higher direct cost. The cost per day to compress an activity is called the "compression cost factor," and is usually expressed in the literature as

$$\text{Compression cost factor} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal duration} - \text{Crash duration}}$$

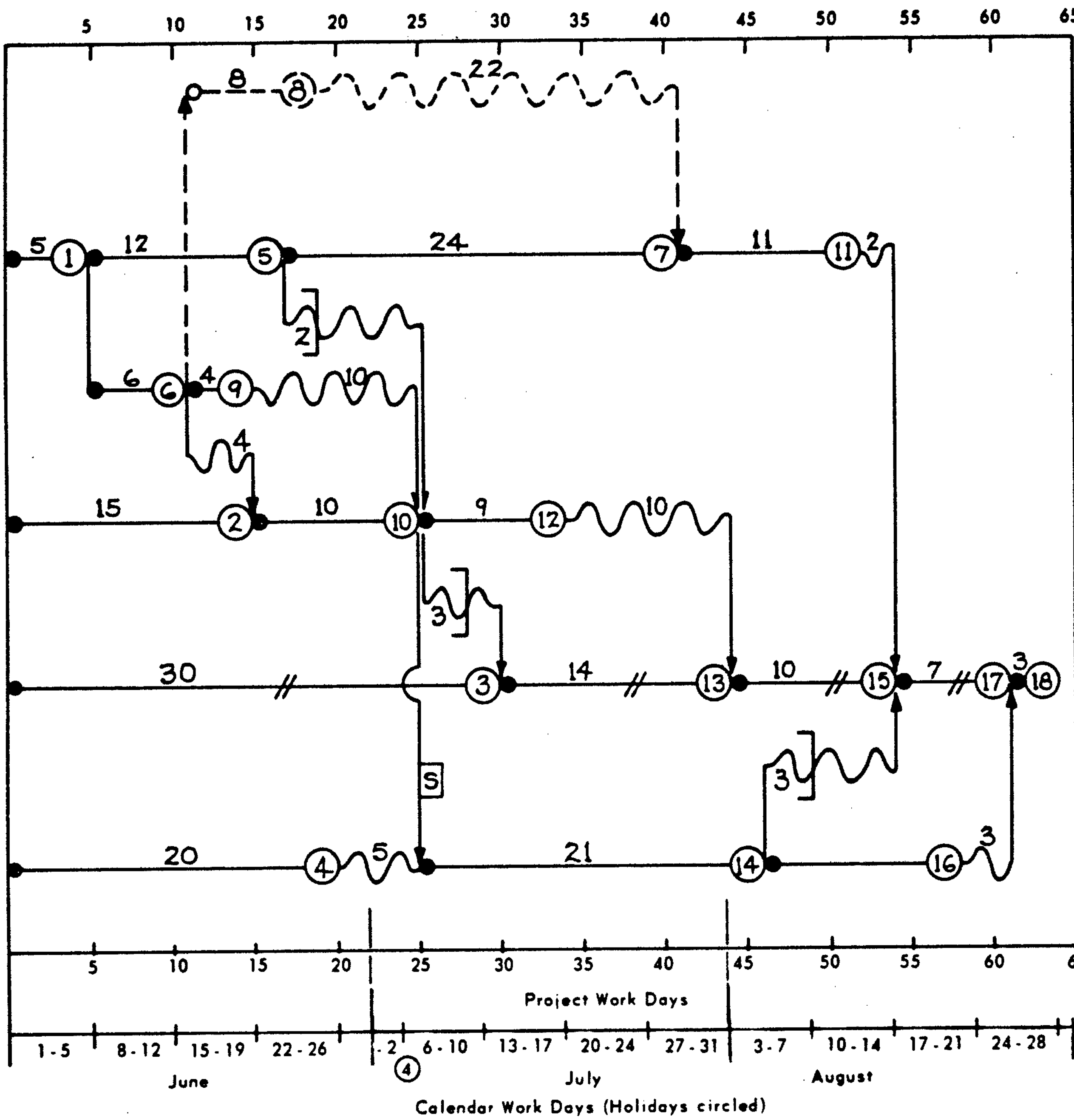


Figure 14. GPA flow diagram schedule for the normal project duration.

Table II. Complete GPA Worksheet

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Description	Department Code	Activity Code	Follows	Normal Duration (days)	Normal Cost ($)	Crash Duration (days)	Crash Cost ($)	Compression Cost Factor ($/day)	Scheduled Duration (days)	Performance Cost ($)	Performance Cost Slope ($/day)
		1	Start 1 June	5	1,500	5		No compression	5 n	1,500	300
Procure		2	Start 1 June	15	7,200	10	8,000	160	15 n	7,200	—
Procure		3	Start 1 June	30	8,400	18	9,000	50	23	8,750	—
Procure		4	Start 1 June	20	2,100	14	2,700	100	20 n	2,100	—
		5	1	12	1,400	8	1,560	40	8 c	1,560	195
		6	1	6	800	4	1,200	200	6 n	800	133
		7	5	24	6,800	20	7,800	250	23	7,050	306
		8_g	6	8	1,000	5	1,240	80	8 n	1,000	125
		9	6	4	600	3	900	300	4 n	600	150
		10	2,6	10	3,000	7	3,450	150	8	3,300	413
		11	7,8	11	2,500	8	3,580	360	11 n	2,500	227
		12	5,9,10	9	1,800	6	2,700	300	9 n	1,800	200
		13	3,10	14	2,600	10	3,320	180	14 n	2,600	186
		14	4,12(s)	21	2,100	15	3,000	150	19	2,400	126
		15	12,13	10	840	6	1,080	60	10 n	840	84
		16	14	12	1,900	10	2,140	120	10 c	2,140	214
		17	11,14,15	7	1,300	5	1,400	50	5 c	1,400	280
		18	16,17	3	700	2	840	140	2 c	840	420
					46,540		55,410			48,380	

Note: The data shown in columns 10 – 12 are for a project acceleration of 10 days.
(g) = ghost; (s) = complex concurrency; n = normal duration; c = crash duration.

Some readers will recognize that this expression is linear — it implies that the cost to compress an activity day by day is constant over the range of possible durations. Although this point of view introduces some degree of error, it is used for the sake of workability. (In any event, the error involved by the assumption of linearity is probably no more serious than the error associated with standard estimating practices.) By completing the "Compression Cost Factor" column on the project worksheet, the planner has a comparative measure of how much it will cost to compress each activity per day. These cost rates will be a determining factor in deciding which activities to compress when accelerating a project. To illustrate this point and introduce another important concept, assume that the project shown in Figure 14 must be accelerated. The critical path (3-13-15-17-18), which is the key to the project duration, must be shortened — and it should be shortened at the lowest cost possible. A cost-control diagram of the critical path (Figure 15) is very convenient for this purpose.

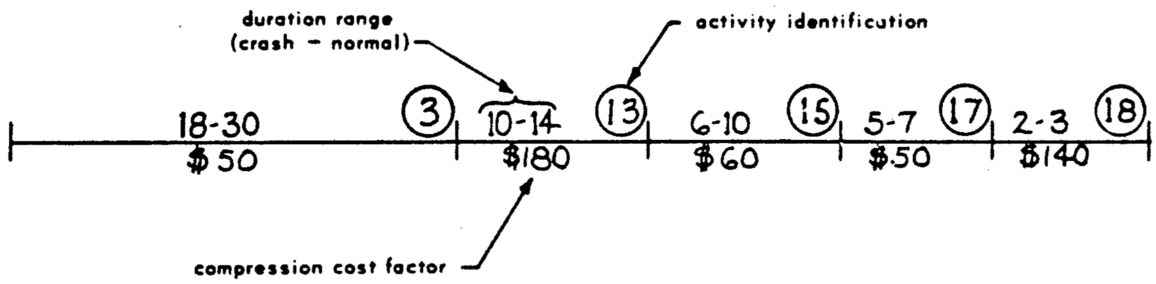


Figure 15. Cost-control diagram for the critical path.

For ease in cross referencing, the general format of the cost-control diagram is similar to that of the flow diagram; the critical path is shown as it appears on the flow diagram — as a straight line in this case. This line is divided into intervals which correspond to the activities which lie on the critical path, but these intervals are not drawn to scale lengths proportional to the time durations of the activities. Other particular features of the cost-control diagram are that an activity's duration range is indicated rather than a specific time duration, and each activity's compression cost factor is shown. A list of activities ranked according to their compression cost factor is called the "order of precedence." When accelerating a project from a schedule of normal durations, an activity has compression precedence over another activity if it costs less to compress. A quick glance at the cost-control diagram (Figure 15) reveals that the critical path can be compressed at lowest cost by compressing activities 3 and/or 17, both of which cost $50 per day to compress.

However, once these activities have been compressed to their crash durations, it logically follows that any further compression would occur in order of precedence: activity 15, then 18, then 13.

Now that activity precedence and the cost-control diagram have been introduced, the implementation of GPA/Cost procedures might best be understood by demonstration, so let us specify that the project at hand must be accelerated by 5 days (one working week). Activities 3 and 17 have equal precedence, but for the sake of making a decision let us compress activity 3 by all of the 5 days necessary; activities 13, 15, 17, and 18 will remain at their normal durations. This allocation of compression will be tested later on, and, if necessary, corrected. The assigned duration times are entered on the cost-control diagram just above the duration ranges for each activity:

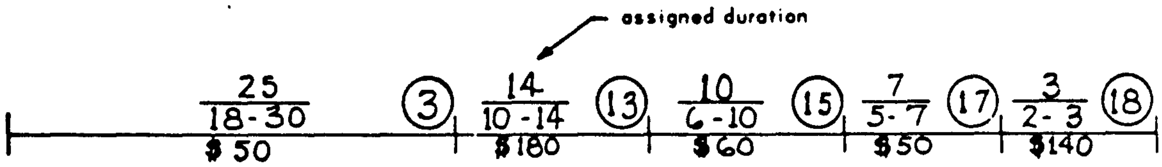


Figure 16. Cost-control diagram with durations assigned to the activities.

Once the critical path has been compressed, it provides a reference for compressing the other paths in the GPA network. These paths are most easily accounted for one at a time on the cost-control diagram in order of decreasing criticality. The order of criticality is determined by the number of days of schedule latitude which occurs in each path. For example, referring to the GPA flow diagram (Figure 14), the next path which should be considered is 1-5-7-11-17-18, which has the least schedule latitude. Note (Figure 17) that the general format of the GPA flow diagram is maintained when this path is added to the cost-control diagram. The point at which the two paths join is called a "node."

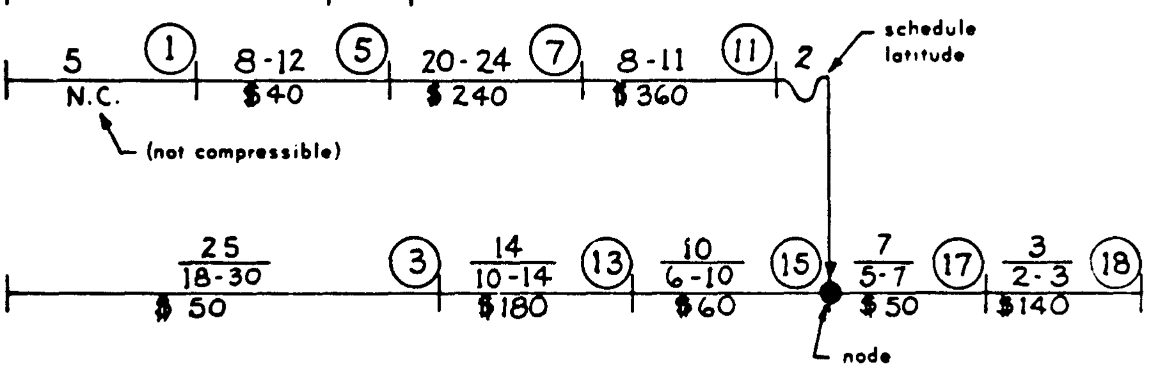


Figure 17. Cost-control diagram with a second path added in order of decreasing criticality.

Now, the new path must be compressed to fit the compressed critical path; this process is called "path justification." Path 1-5-7-11-17-18 contains 2 days of schedule latitude, thus it can be compressed by 2 days without affecting the duration of any activities on the path — the schedule latitude is simply absorbed. But this path, which is now critical, must be accelerated by a total of 5 days, as has been decided, so 3 days of activity compression are necessary. Activity 1 cannot be compressed, as determined on the project worksheet, and it is clear that activity 5 has the highest precedence — it costs the least to compress. Thus activity 5 should be compressed by the 3 days required. At this point, both paths have been compressed to satisfy the project acceleration of 5 days, and both paths separately have met the requirement of least-cost compression. However, the interaction of these paths raises the question of whether or not least-cost compression for the paths considered together has actually occurred. The question is simply: would it cost less to compress activity 17 or 18 which occur on both paths instead of compressing activities 3 and 5 which lie separately? A project acceleration of 5 days could still be satisfied by compressing activity 17 (which has a higher precedence than activity 18) and correspondingly extending activities 3 and 5. The answer to the question is found in a simple cost investigation involving activities to the right and left of the node. It would cost $50 per day to compress activity 17, and the total saving of extending activities 3 and 5 is $90 per day, which would result in a net saving of $40 per day. Therefore, to achieve a true least cost for the coupled paths, activity 17 should be compressed by 2 days up to its crash duration and activities 3 and 5 should be extended by 2 days. Would the compression of activity 18 by 1 day up to its crash duration and the extension of activities 3 and 5 by still another day further reduce cost? No — there would be a net loss of $50, as it would cost $140 to compress activity 18 and the total saving of activities 3 and 5 would be only $90. The assignment of activity durations for the second path, with corrections for activities 3 and 17, would be as shown in Figure 18.

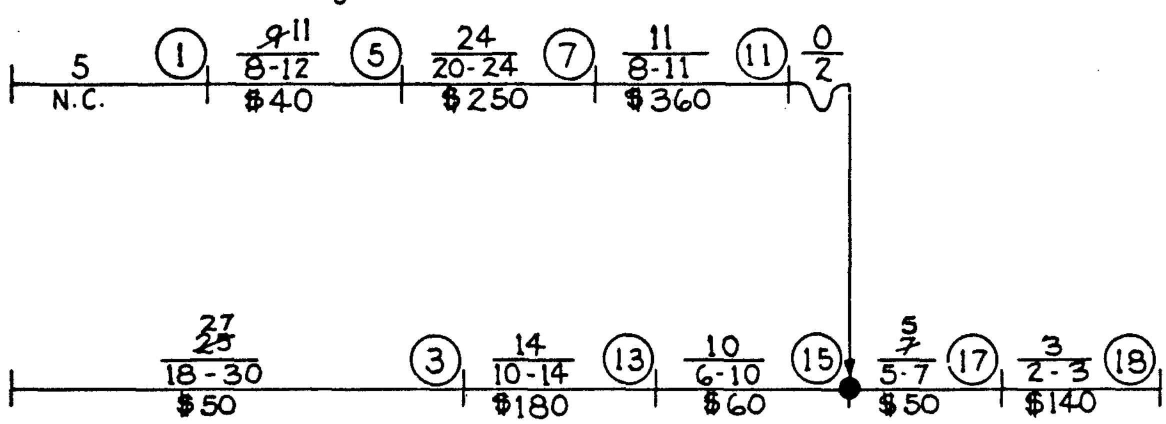


Figure 18. Activity durations have been assigned to the second path.

Now add the next path in order of decreasing criticality. Using the GPA flow diagram, this path is found to be 2-10-14-16-18 with 3 days of schedule latitude. Clearly this path can be compressed 3 days by the absorption of schedule latitude without affecting any activity durations. However, at this point the path becomes critical, and 2 days of activity compression are required to complete path justification (5 days of total path compression). According to precedence, activity 16 should be compressed by 2 days up to its crash duration. This move satisfies the compression requirements, but does not necessarily prove that lowest costs have been attained, because the path is coupled to the previously considered paths at the node between activities 16 and 18. A cost investigation is necessary. — Would it be advisable to compress activity 18 (common to all three paths) up to its crash duration and extend activities 3, 5, and 16 by 1 day? The reader should assure himself that this move would still satisfy the 5-day compression requirement. It would cost $140 to compress activity 18, but a total of $210 would be saved by extending activities 3, 5, and 16. The net saving is $70, therefore this move is advisable (Figure 19). Further compression of activity 18, which is the only common activity on the new path, is impossible, consequently the goal of lowest cost has been achieved.

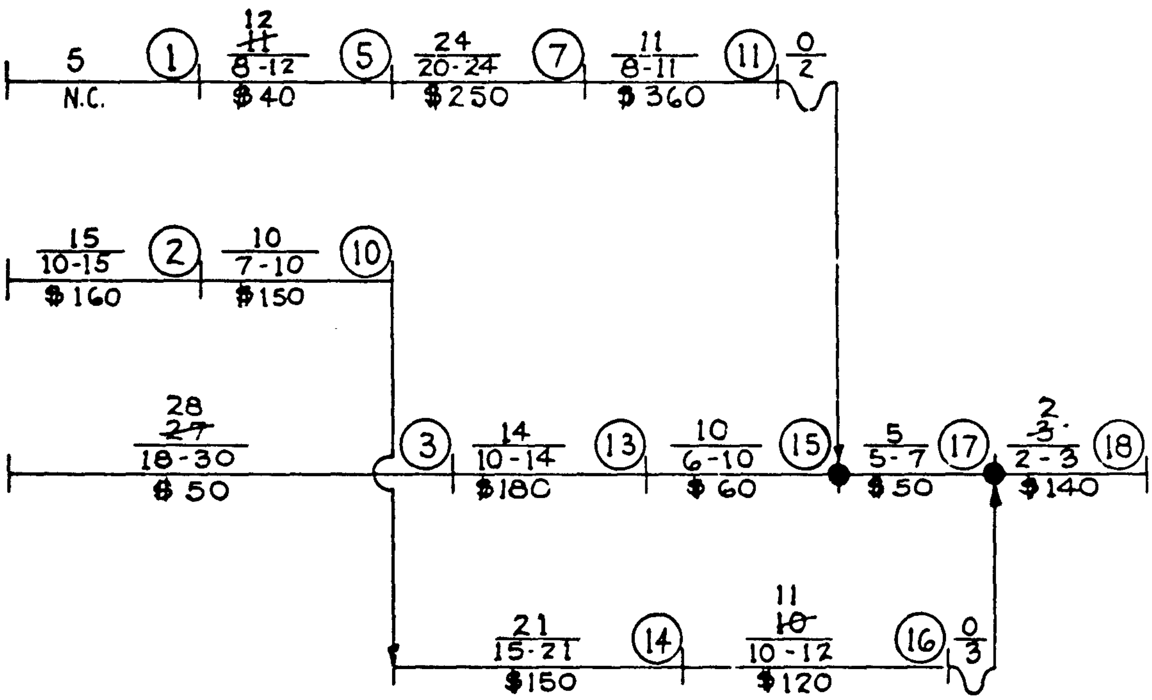


Figure 19. A third path is added.

Referring once again to the GPA flow diagram, the next path in order of criticality is 2-10-13-15-17-18 with 5 days of schedule latitude. There are two aspects of this path which have not been encountered before. First of all, taken by itself it contains enough schedule latitude so that it could be compressed by all 5 days required for path justification without affecting any activity durations — but the path would become critical. Second, all of the activities have already been considered on the cost-control diagram. Although this new path cannot be ignored, all that needs to be done is to determine if the link between activities 10 and 13 can be fitted into the network now standing on the cost-control diagram. Examination of path 2-10-13-15-17-18 shows that activities 17 and 18 have already been compressed by a total of 3 days, thus the balance of the required 5-day path compression can be absorbed by the schedule latitude comprising the link between "10" and "13." The reader should find that this link is possible, and that it consists of 3 days of latitude. The presence of schedule latitude in path 2-10-13-15-17-18 may be confusing at first because all the activities have already been determined to be critical on other paths. However, this latitude is unavailable and is shown (Figure 20) only to indicate activity interdependence. One other point which should be mentioned is that a cost investigation at the coupling node is unnecessary because none of the activity durations were changed to fit this path into the existing network on the cost-control diagram.

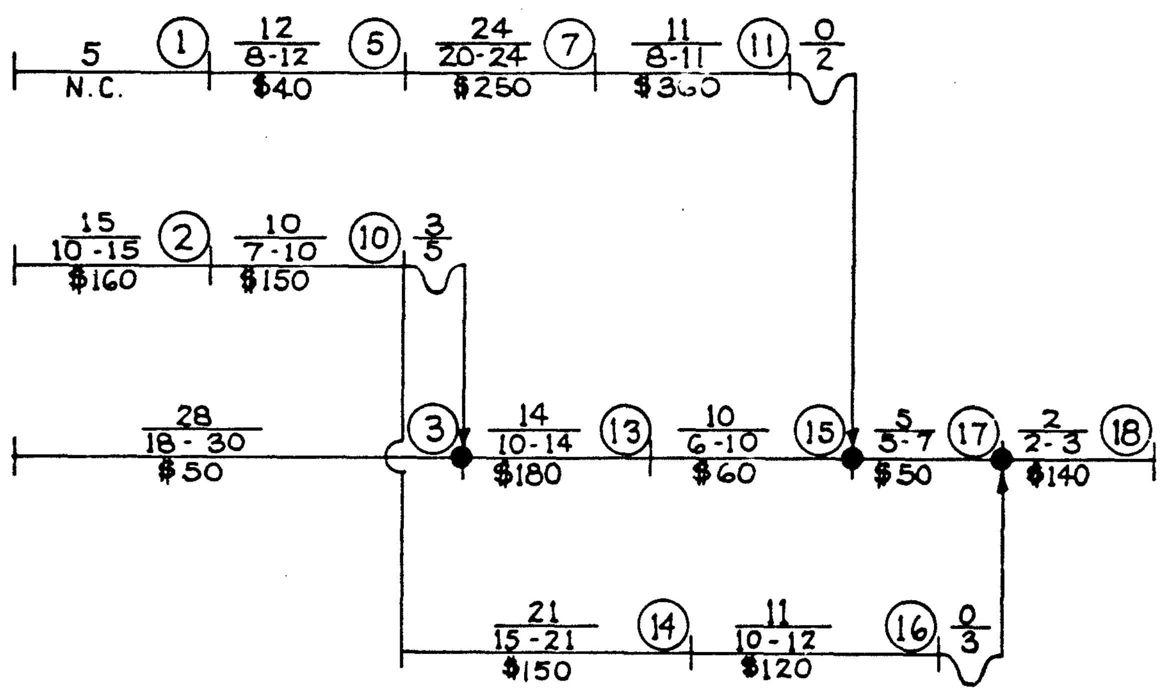


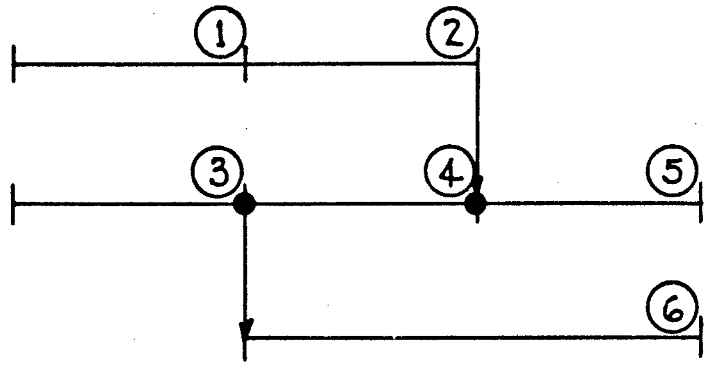
Figure 20. The completed cost-control diagram.

To continue with the cost scheduling of the project, the GPA flow diagram (Figure 14) shows that the remaining paths contain more than 5 days of schedule latitude. All of these paths can be compressed by the required amount without becoming critical; consequently activities 4, 6, 8_g, 9, and 12 which have not yet been considered on the cost-control diagram can all be scheduled at their normal durations. The total direct-cost increase to accelerate the project by 5 days is $460 (see "Determination of Project Direct Costs").

A GPA flow diagram can be drawn up very quickly for the accelerated project duration (using the activity durations derived on the cost-control diagram) because the new flow diagram will closely resemble the flow diagram for the normal project duration. The use of activity vector strips is not necessary — the activity vectors, as well as the dependence arrows, schedule-latitude symbols, etc., can be drawn directly on a new diagram blank. But changes to the new schedule can be made more easily if vector strips are used.

Now let us summarize the GPA/Cost procedures demonstrated above and reflect upon the concepts which lie behind them, so that better insight might be gained into project acceleration at lowest cost. To begin with, paths containing schedule latitude less than or equal to the amount of project acceleration are added to the cost-control diagram one at a time in order of decreasing criticality. Each path added is compressed by the absorption of schedule latitude and by the compression of activities according to precedence until it satisfies the established project duration within the constraints of paths which may have already been considered. If this process, called "path justification," results in the new path becoming critical, a cost investigation is conducted to assure the attainment of lowest possible costs. This investigation simultaneously considers all paths coupled on the cost-control diagram. The cost investigation is based on the supposition that the compression of an activity common to two or more paths might cost less than the compression of activities which occur separately. Thus, when conducting a cost investigation, first look for compressible activities which are common to the new path and paths already considered. If all the common activities are already scheduled at their crash durations, no cost investigation is required because activities were compressed on the basis of precedence during path justification. If, on the other hand, compression of common activities is possible, propose the compression of the activity having the highest precedence. Now find separate activities on the other side of the node which would be extended to satisfy the requirements of path compression. If several activities on a path can be extended, choose the one having the highest compression cost factor (lowest compression precedence).* In certain cases, path interactions may result in a series of activity duration changes; for example:

* Some planners may prefer to consider extension precedence (ranking of activities in order of decreasing cost to compress) so that the extension of activities according to precedence is consistent with the compression of activities.



Assume that the proposed compression of activity 5 would result in the extension of activity 1 and 3. The extension of activity 3 would then require the compression of activity 6.

Once all the proposed activity compressions and extensions have been made, the cost factors of the activities affected are totaled. When activities have been compressed, the cost to compress is considered as a cost, but when activities have been extended, the cost to compress is thought of as a saving. If a net loss occurs, the paths are coupled at lowest cost as they stand. A net saving indicates that the proposed activity compressions and extensions are advisable. The common activity first considered should be compressed until it, or one of the other activities affected, reaches the limit of its duration range. When this limit is reached, the activity on the path next in order of precedence (on the same side of the node) is substituted; costs and savings are totaled again, and a comparison is made as before to determine if further duration changes are advisable.

At this point the reader is urged to apply GPA/Cost scheduling methods to the sample project accelerated by 10 days. The activity durations which should result are listed in column 10 of Table II; a step-by-step solution is given in Appendix C. In this exercise, the reader will find that the need for conducting cost investigations is sharply reduced. It has generally been found that as a schedule approaches its crash limit, the cost investigation possibilities usually diminish because more activities are compressed to their crash durations when the paths are justified. In other words, the paths become less flexible.

Project Acceleration to the Crash Duration

The determination of the project's crash duration, which will indicate how much compress is necessary, can be made easily if an intermediate amount of project compression has already been carried out on a cost-control diagram. For example, turn back to the completed cost-control diagram for the 5-day project acceleration (Figure 20). Now, using crash durations assigned to the activities, find the crash duration of each of the three critical paths. The longest path is 1-5-7-11-17-18, which is 48 days long. This path limits the acceleration of the project to a duration of 48 days, and therefore it is the basic critical path for the project at its crash duration (Figure 21). The flow diagram schedule for the normal duration (Figure 14) shows that the normal project duration is 64 days; thus the amount of compression necessary to reach the project's crash duration from the normal duration is (64 - 48) = 16 days. Although this method is not infallible, it usually works because the paths which become critical for intermediate amounts of project acceleration almost always include the path which ultimately limits acceleration of the project. However, occasionally the limiting path develops beyond the intermediate duration point considered. This problem will become apparent when a new path cannot be fitted into the cost-control diagram network even though it has been compressed to its crash duration. It is now assumed that the crash duration of the project is determined by the duration of this new fully compressed path, and the cost-control diagram is revised accordingly.

A direct approach toward finding the least-cost project schedule for the crash duration is to develop an overaccelerated GPA flow diagram in which all activities are shown at their crash durations, and then extend noncritical activities to minimize cost. This alternate technique, explained in Appendix D, should be used whenever the project must be crashed.

Determination of Project Direct Costs

The total direct cost of a project for the normal duration can be determined by simply totaling the normal activity costs entered on the project worksheet. Similarly, the cost of crashing all activities can be found by totaling the activity crash costs. The project's lowest direct cost for the crash duration and intermediate durations is derived from the respective cost-control diagrams. Activities scheduled at their normal or crash durations are assigned those costs respectively. The cost of activities which have been partially compressed is calculated as follows: First determine how many days or other units of time the activity has been compressed; multiply this number by the activity's compression cost factor; and then add the result to the normal cost.

Activity cost = Normal cost + (Days of compression x Compression cost factor)

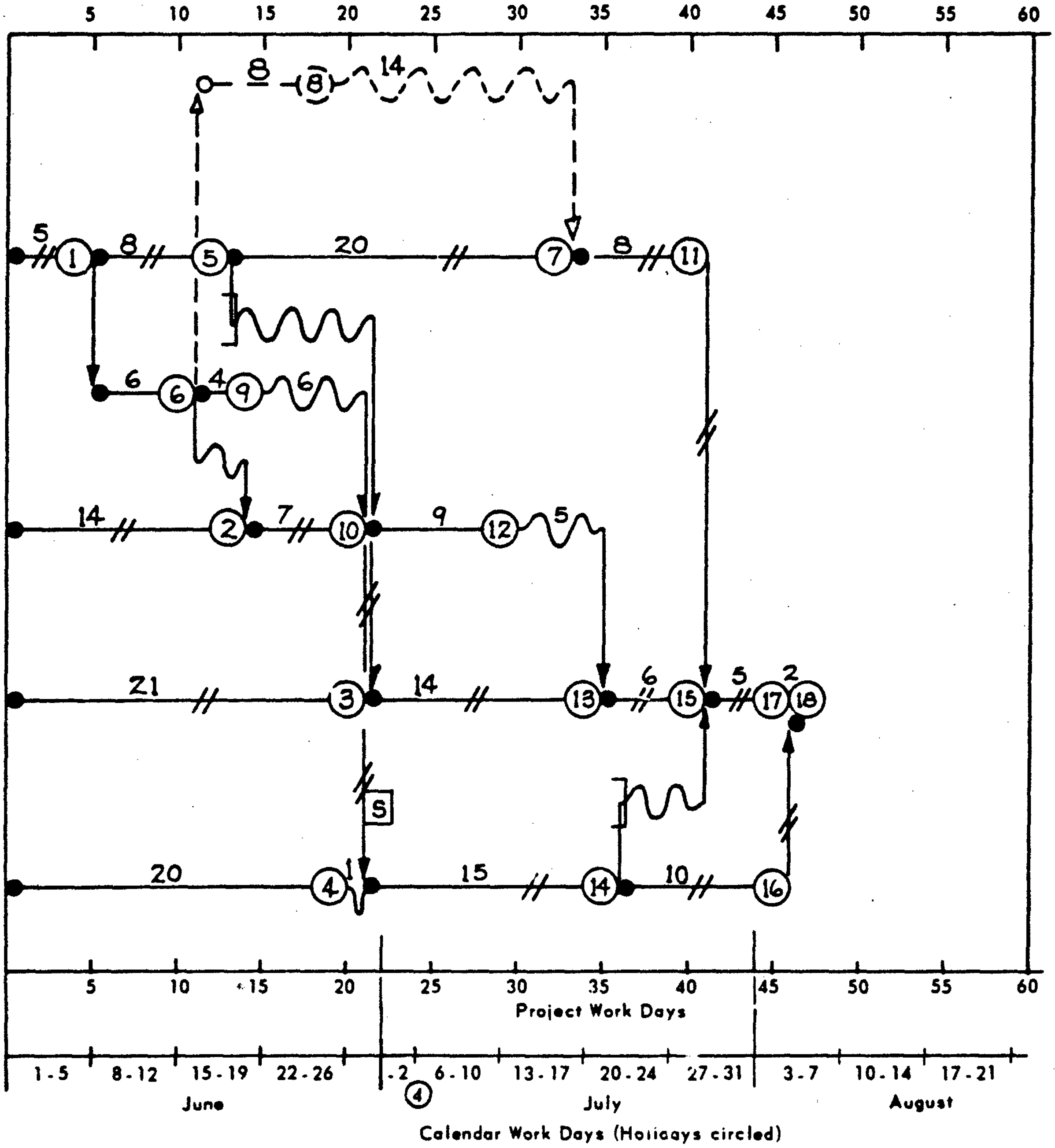


Figure 21. GPA flow diagram schedule for the project crashed at least cost.

If activities have been extended from an overaccelerated schedule (Appendix D), it is more convenient to find the activity cost as follows:

Activity cost = Crash cost - (Days of extension x Compression cost factor)

Both methods will yield the same results.

Project Direct Cost as a Function of Duration

The overall significance of project cost scheduling is seen in Figure 22, where direct costs as a function of project duration times have been plotted. The project's direct cost is lowest at its normal duration time, which is consistent with the definition of normal activity duration (see "Intermediate Project Acceleration"). Costs rise at an accelerated rate as the project is compressed up to the crash duration. At this point, costs continue to rise, but the project duration does not decrease any further. The critical crash cost is the lowest cost at which the project can be completed at the crash duration time. In other words, all critical activities have been crashed, but activities on noncritical paths have been selectively compressed to minimize costs. The all-crash point occurs when all activities, critical as well as noncritical, are crashed. All costs between these two points represent varying degrees of unnecessary activity compression. Fondahl[1] has shown (Reference 3, Figure 8) that all possible project costs for the given range of project duration times and activity costs fall within the limits defined by the solid and dashed lines of Figure 22. For each project duration time, there is a range of project direct costs. The merit of GPA/Cost procedures rests on guiding the planner/scheduler to schedule the project at <u>lowest</u> direct cost for a given overall duration.

Determination of Lowest Total Project Cost

The project duration versus direct cost plot can be coupled with indirect costs and performance bonus-penalty rates to determine the lowest total project cost (Figure 23).[2] The total cost curve is obtained by simply superimposing the direct, indirect, and performance cost curves at several duration points. Figure 23 shows that the normal project duration is not always associated with the lowest <u>total</u> project cost.

Entry in Worksheet

Once the desired project duration has been established (by lowest total cost or other criteria), a set of activity durations related to the lowest direct cost for that overall duration can be derived on a cost-control diagram. When found, these activity durations are entered in column 10 of the project worksheet, labeled

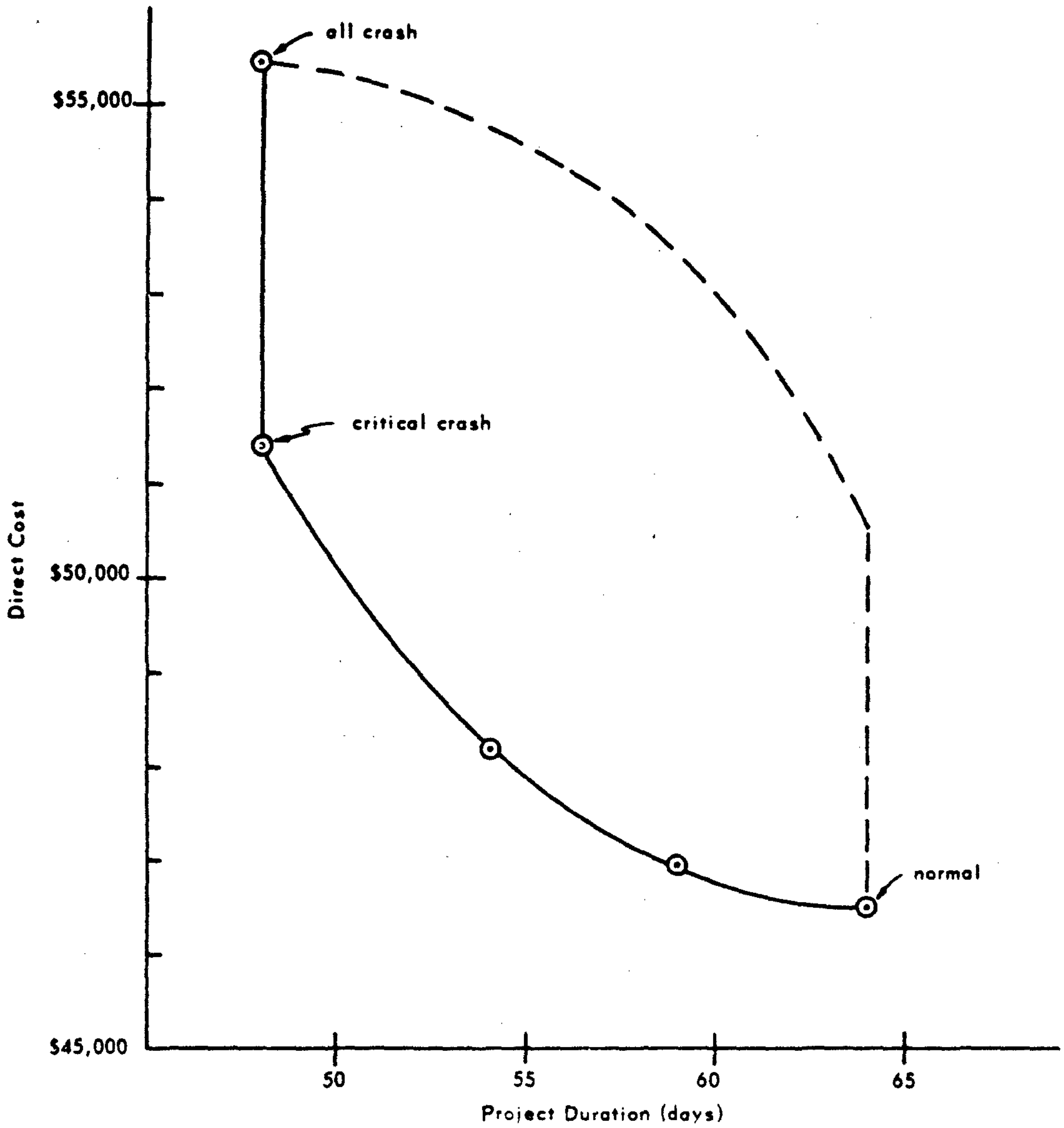


Figure 22. Project direct cost vs duration. (After Fondahl.[3])

"Scheduled Duration." The direct costs associated with the scheduled activity durations are calculated by the methods described under the subject "Determination of Project Direct Costs" and are entered in column 11 of the project worksheet, labeled "Performance Cost." The figures entered on the sample worksheet (Table II) are for the 10-day project acceleration.

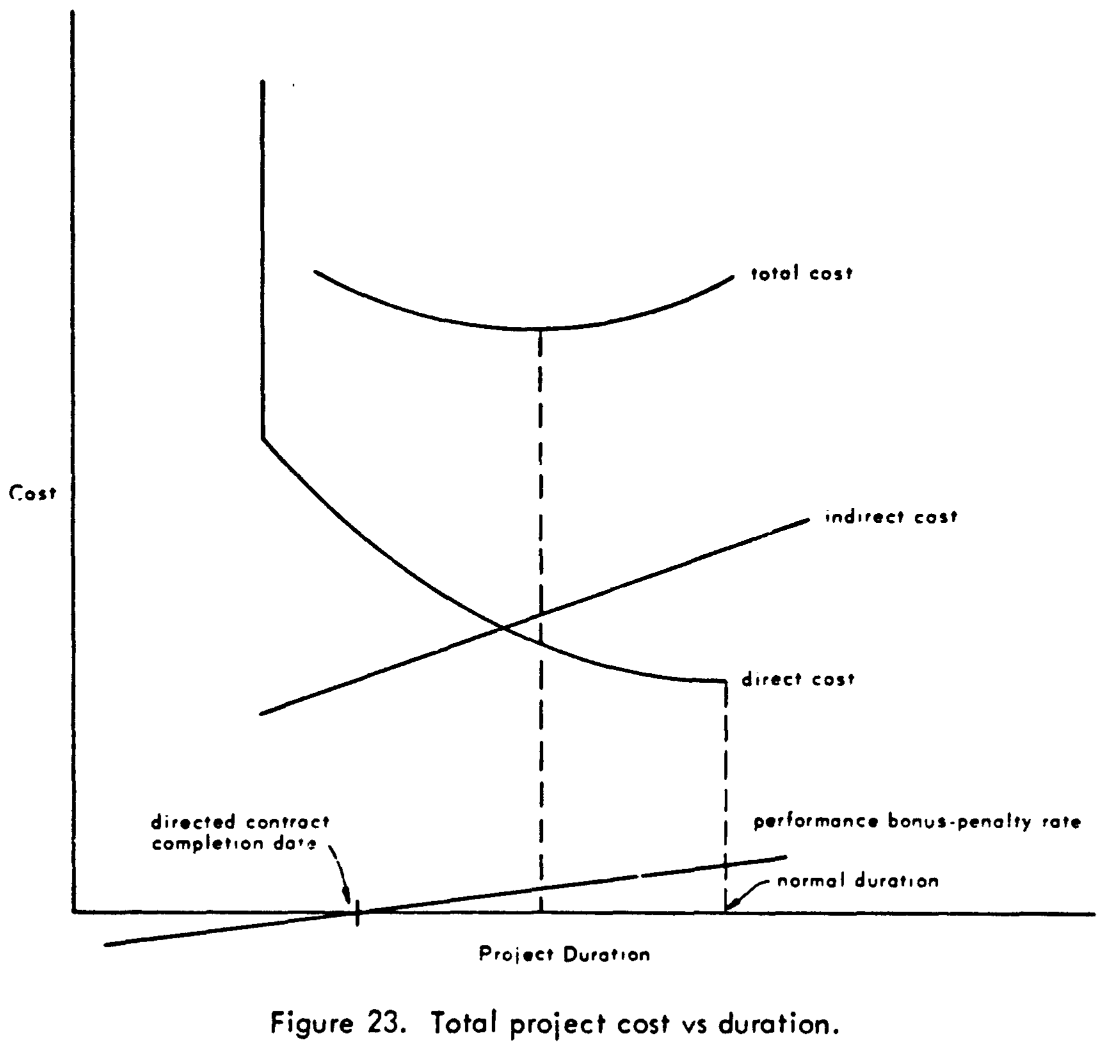


Figure 23. Total project cost vs duration.

SECTION III. DETERMINATION OF CUMULATIVE DIRECT COST AS A FUNCTION OF TIME

Another aspect of GPA/Cost is the development of a project's predicted direct cost curve as a function of time with respect to the activities involved. This curve, which is drawn for a given project duration, is very useful when checking progress against accumulating direct costs to determine if a cost overrun or underrun is occurring. The predicted direct cost curve for the sample project accelerated by 10 days is shown in Figure 24. The significance of the three different curves shown will become apparent as the method used to derive a meaningful graph is discussed. Briefly, this method involves the division of the project duration into a number of time intervals over which direct costs are computed. The costs occurring within a given time interval are accumulated, added to costs accumulated in previous intervals, and the total direct cost is plotted at the end of that interval. When all the time intervals have been considered, the cost points are connected by straight lines to complete the cost curve. But let us consider this procedure in more detail.

The accumulation of costs within a given time interval is basically straightforward. If an activity falls completely within an interval, for example activity 1, its direct cost (entered in column 11 of the project worksheet) is assigned to that interval. However, if an activity lies in more than one interval, for example activity 5, the direct costs must be apportioned to the respective intervals according to the amounts of duration occurring in each interval. The procedure of cost distribution requires that the activity's performance cost slope be found:

$$\text{Performance cost slope} = \frac{\text{Activity direct cost for the scheduled duration}}{\text{Scheduled duration}}$$

The performance cost slope is multiplied by that portion of the activity's duration which occurs within a given interval to determine the apportioned cost which is assigned to the interval. Consider activity 5, which lies in both the first and second intervals. The performance cost slope of activity 5 is $195 per day. Five days of activity 5 occur in the first interval, thus the cost which is assigned to the first interval is $195/day x 5 days, or $975. The balance of the activity's direct cost is assigned to the second interval.

One common exception to the apportionment of direct costs arises when the activity is a procurement, for example activity 2. In this case, the total direct cost is often assigned to the interval within which the activity is started (the first interval for activity 2) because the total cost is obligated when the purchase order or contract has been executed. An alternative is to assign the total procurement cost to the time interval within which the invoice is expected.

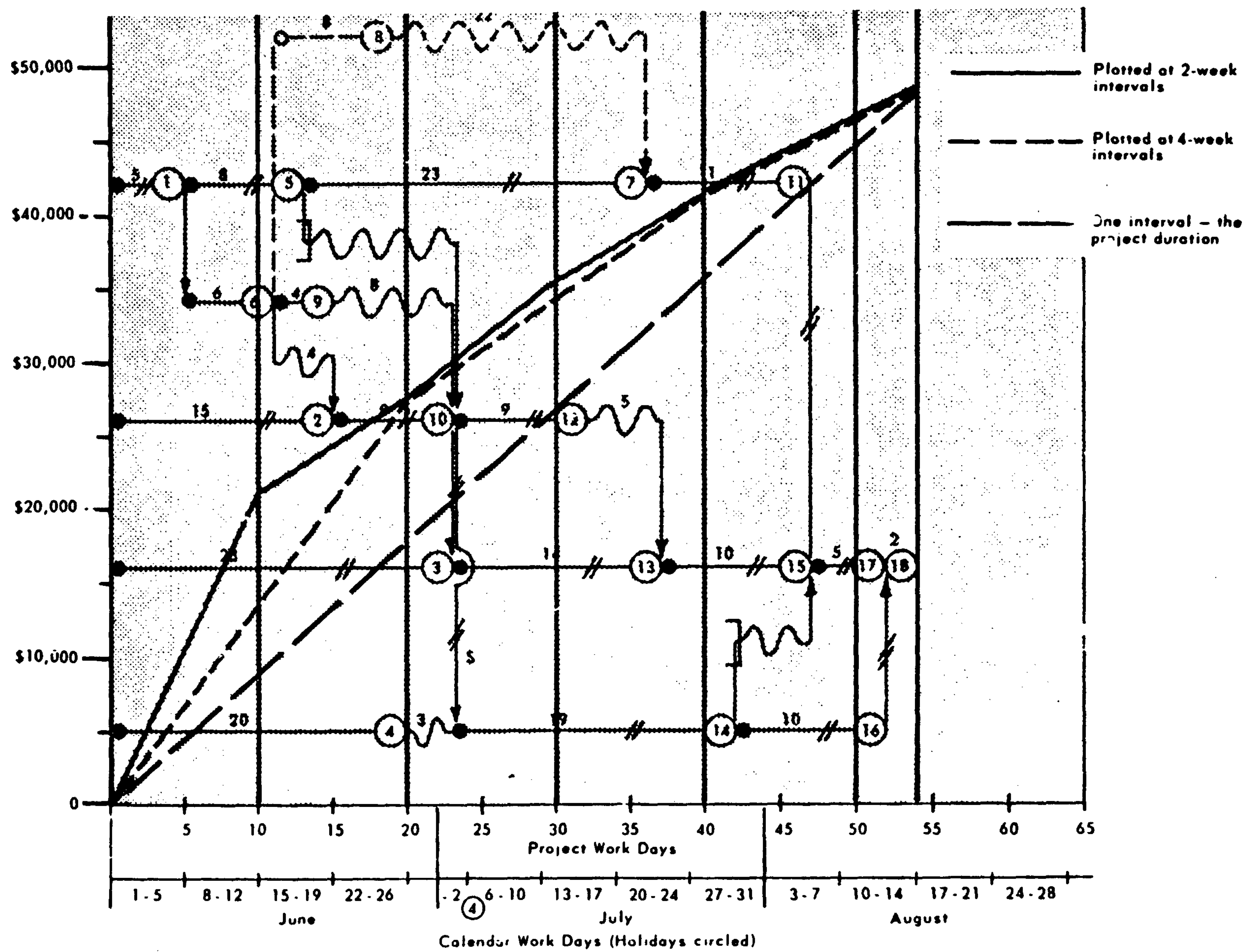


Figure 24. Cumulative direct costs plotted over the project duration.

As mentioned earlier, once the direct costs which occur within a given time interval have been determined, they are totaled and superposed over costs accumulated in previous intervals. The final result is then plotted at the end of the interval being considered. For example, considering the first interval (Figure 24), the accumulated costs are plotted at June 12 — there is no previous cost interval. The costs occurring within the second interval are added to the costs accumulated during the first interval, and the total cost is plotted at June 26. When all the intervals have been considered in this manner, the plotted points are connected by straight-line segments, each of which is a linear approximation of the project's direct cost curve over the respective interval. Accordingly, the complete cost curve consists of a series of straight-line approximations over all the time intervals considered.

The fact that the predicted direct cost curve is an approximation leads us to investigate the accuracy of such a curve. Within the limitations imposed by linear cost slopes, the accuracy of the cost curve depends upon the number of time intervals considered. The higher the number of intervals, the greater the accuracy — but the amount of effort required to develop the cost curve also increases. The three cost curves in Figure 24 illustrate this point. The straight line comprised of short dashes was drawn by simply connecting the total project cost occurring on the last working day with the zero point at the beginning of the first working day. In this case, the time interval is the entire project duration. The line consisting of long dashes resulted when the project duration was divided into three 4-week intervals. When the project was divided into six time intervals of 2 weeks each, the solid line resulted. Clearly there is a significant difference between the straight-line cost curve and the more accurate solid-line curve. Assuming that the project progresses as scheduled and that incurred direct costs agree with the estimated costs, the straight-line plot could be misleading because one would believe that a serious direct-cost overrun was occurring when actually the costs were following the predicted path shown by the more accurate solid-line curve. The amount of accuracy required depends upon many factors, thus it would be quite impractical to try to develop a criterion which could be applied to all projects and situations. Nevertheless, there is one general guideline: time intervals over which direct costs are accumulated should correspond to the spans of time between project cost-checks. Following this guideline will result in a timely and meaningful basis for direct-cost comparisons when cost-check intervals have been established. Note that this criterion does not set forth the proper frequency of cost-checks which should be made for a given project. The frequency of cost-checks ultimately rests on the amount of cost feedback required by management.

When conducting a direct cost-check, it is essential that actual direct costs be analyzed in light of actual progress. Therefore, it is suggested that as the project progresses, a colored line be drawn parallel to each activity pursuit line to indicate the percentage of work accomplished on each activity. Then when a cost-check cut-off date has been reached, and progress has not occurred according to schedule, a target cost can be computed as follows:

1. First consider all activities completed by the cutoff date and total their direct costs. Include activities completed ahead of schedule.

2. Next consider activities which have been partially completed at the cutoff date, including activities which have been started ahead of schedule. Measure the length of the percentage-of-completion line associated with each activity using the flow diagram scale regardless of how much time was required to reach that point of partial completion. For example, if an activity was originally scheduled to be performed within 10 days, and at the end of 10 days only 50 percent of the activity was actually accomplished, the activity would be measured at and charged with only 5 days of effort for purposes of calculating the target cost. On the other hand, if the activity had been 90 percent complete within 5 days, it would be charged with 9 days of effort. Now multiply the number of days charged to each partially completed activity by its respective performance cost slope to find its target cost.

3. Add the activity target costs found in steps 1 and 2 to find the total target cost for the partially completed project. The target cost thus found represents the estimated direct cost of the progress achieved by the cutoff date; it is a revision of the project's predicted direct cost curve which accounts for the fact that the schedule has not been adhered to.

4. To complete the cost-check, direct costs actually incurred are compared with the total target cost to determine if there is a cost overrun or underrun. Whereas the percentage-of-completion lines give a clear picture of progress in terms of the project's operational aspects, the target cost, when compared to the predicted cost, gives a measure of progress in terms of cost. All of this information can be of great value to management when evaluating project progress.

SECTION IV. CONCLUDING REMARKS

Experience has shown the Graphical PERT Analog to be an effective approach to the planning, scheduling, and progress review of moderately complex projects. Although projects including as many as 150 activities have been analyzed by GPA methods, there is a point at which project programming can be accomplished more efficiently with a digital computer. This point is usually determined by the complexity of activity interactions rather than the number of activities, thus it is not clearly defined for all situations. The question of project complexity also arises when one attempts to define the cutoff point between formal programming and "just letting the job unfold." Here again, conditions and requirements vary, and each user must determine his own specific needs.

The scope of GPA includes three basic units which can be selectively employed to suit the requirements of management control for various projects. The development of the flow diagram schedule is directed toward the operational aspects of the project.

If the directed completion date occurs before the "normal" completion date (as determined by GPA), and this directed date must be met, then GPA cost scheduling procedures can be used to accelerate the project by the required amount at lowest direct cost. If, on the other hand, the directed completion date is just a target date, one can carry the cost analysis further to find the project completion date associated with the lowest total cost, and then schedule to minimize direct costs. When management needs cost information in relation to project progress, the direct cost curve as a function of time can be developed. It is important to recognize that there is no universal criterion which governs the use of GPA because the extent to which these techniques are applied must depend upon the operational and economic advantages afforded to meet the individual requirements of each user.

ACKNOWLEDGMENTS

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Appendix A

EXERCISE IN CONSTRUCTING AND INTERPRETING A GPA FLOW DIAGRAM

This exercise is designed to provide practice in analyzing a project using the GPA techniques described in Section I. The completed worksheet is shown in Table A-I, and the flow diagram blank as well as the set of activity vectors have been prepared for convenience (Figures A-1, A-2). It is suggested that the activity vectors and flow diagram blank be reproduced or traced on separate pieces of paper so that this report may be left intact. A correct flow diagram solution does not have to follow a particular format — provided all the activities are properly constrained within the framework of the logic sequence. The suggested format for easy readability is shown in Figure 14 of the main text. Test your solution by answering the following questions:

1. According to the GPA schedule, how many days are required to complete this project?

2. What is the critical path?

3. What is the effect of shortening activity 15 by 2 days?

4. Is the ghost activity 8_g a potential problem?

5. If the work remains on schedule, by what date will management know if ghost activity 8_g is necessary?

6. Assuming that activities 8_g and 10 require the same piece of equipment for execution, and there are no other restrictions, when would you recommend that 8_g be started? How many days of schedule latitude would be available?

7. Find three cases of unavailable schedule latitude, identify them by activity paths (including the restrictive path) and note if there is any unshared or shared schedule latitude associated with these paths.

<u>Answers</u>

1. 64 days. The project should be completed by August 27.

2. Path 3-13-15-17-18.

3. (a) The critical path is shortened by 2 days.

   (b) A second critical path is created: 1-5-7-11-17-18. Both paths are equally important in terms of scheduling.

   (c) A potential problem path is created having only 1 day of schedule latitude: 2-10-12(s)-14-16-18.

4. Probably not, because there are 22 working days of unshared schedule latitude associated with activity 8_g. In most cases this amount of schedule latitude would be sufficient to allow enough freedom for routine resource reallocation.

5. June 15.

6. Start activity 8_g on July 6. There would still be 8 days of unshared schedule latitude available.

7. (a) Path ...5 $\overset{8}{\text{-------}}$ 12 in relation to path ...5-7-11 $\overset{2}{\text{-------}}$ 17. Delaying completion of activity 5 by more than two days will interfere with the start of activity 17. Path ...5 $\overset{8}{\text{-------}}$ 12 consists of 2 days of shared schedule latitude and 6 days of unavailable latitude.

   (b) Path ...14 $\overset{8}{\text{-------}}$ 17 in relation to path ...14-16 $\overset{3}{\text{-------}}$ 18. Path ...14 $\overset{8}{\text{-------}}$ 17 consists of 3 days of shared schedule latitude and 5 days of unavailable latitude.

   (c) Path ...10 $\overset{5}{\text{-------}}$ 13 in relation to path ...10-12(s)-14-16 $\overset{3}{\text{-------}}$ 18. Path ...10 $\overset{5}{\text{-------}}$ 13 consists of 3 days of shared schedule latitude and 2 days of unavailable latitude.

Table A-1. Basic GPA Worksheet for the Exercise Problem

Description	Department Code	Activity Code	Follows	Normal Duration (days)
		1	Start 1 June	5
Procure		2	Start 1 June	15
Procure		3	Start 1 June	30
Procure		4	Start 1 June	20
		5	1	12
		6	1	6
		7	5	24
		8_g	6	8
		9	6	4
		10	2,6	10
		11	7,8	11
		12	5,9,10	9
		13	3,10	14
		14	4,12(s)	21
		15	12,13	10
		16	14	12
		17	11,14,15	7
		18	16,17	3

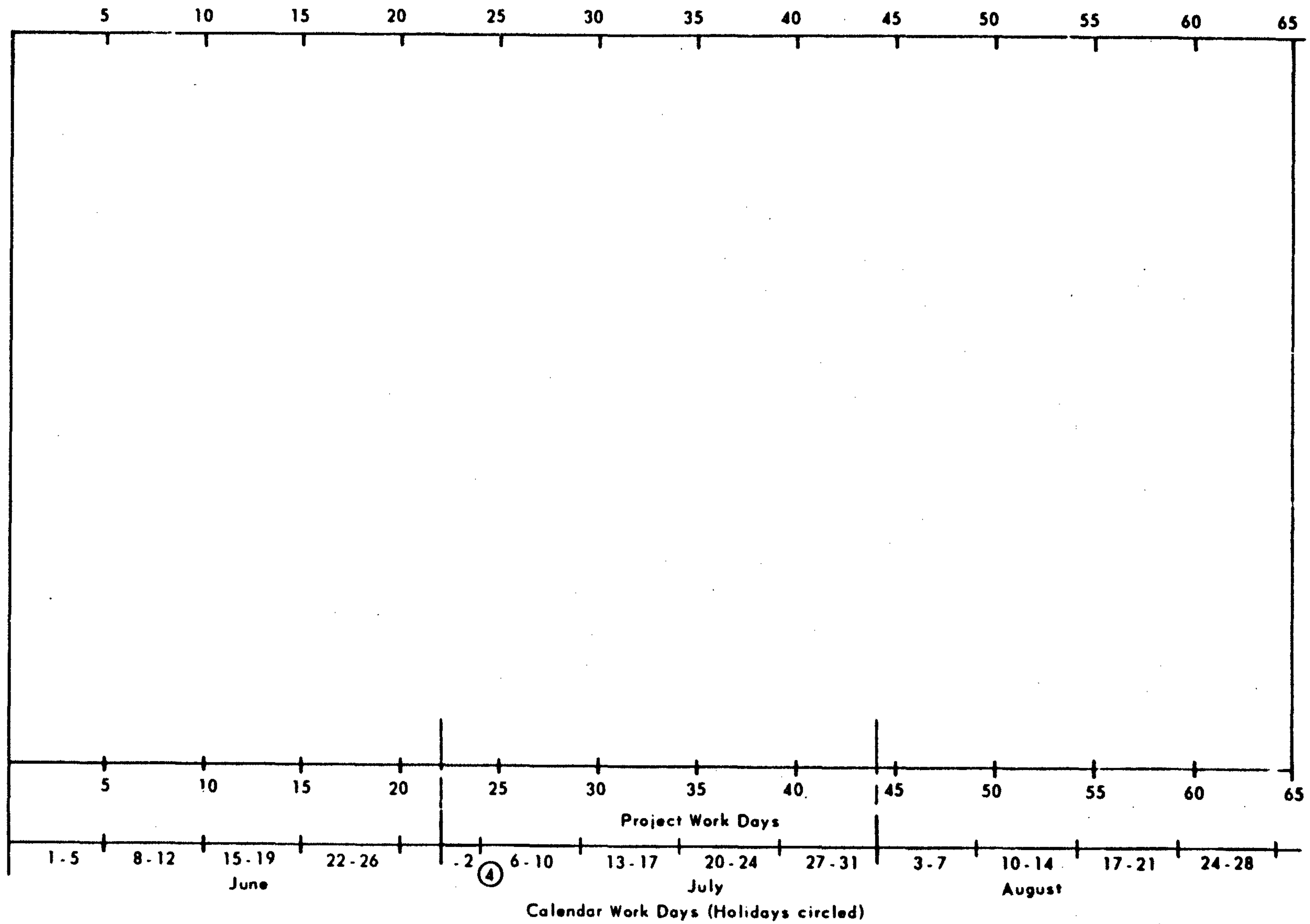

Figure A-1. GPA flow diagram blank for the exercise problem.

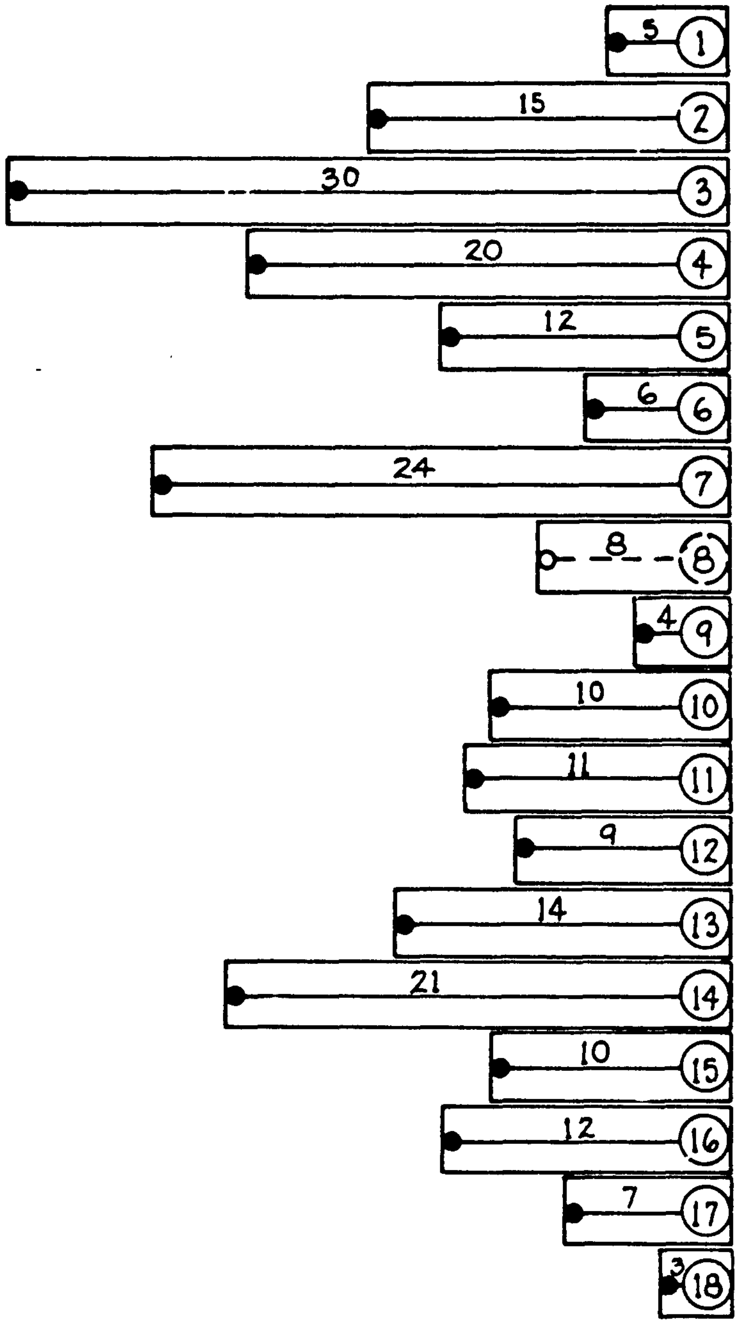

Figure A-2. Array of activity vectors for the exercise problem.

Appendix B

METHOD FOR OBTAINING A REPRODUCIBLE GPA FLOW DIAGRAM

The following technique allows the GPA flow diagram to be reproduced by standard dry reproduction equipment and reduces the total planning time if a number of projects are to be programmed.

1. Draw up a master GPA diagram blank on tracing paper or Mylar* using India ink, or pencil and carbon backup paper, for maximum contrast. The size and scale of the blank should be as large as practicable. Once made, this master diagram blank can be used to print any number of working diagram blanks.

2. Make sepia prints of the master diagram blank. For best results the following precautions should be observed:

   (a) Maximum readability of the final schedule will be attained if the printing process is carried out slowly enough to "burn out" the background of the sepia paper.

   (b) Because of the dimensional instability of sepia paper to heat and strain, the sepia paper and master diagram blank should be fed widthwise instead of lengthwise. This procedure will minimize distortion of the time scale due to stretching of the sepia print during processing.

3. Draw the various activity vectors on tracing paper with a soft pencil to obtain maximum contrast. The use of carbon backup paper will be helpful in this respect. India ink may be used instead of a pencil (and backup paper).

4. Now proceed in the usual manner to complete a master GPA schedule using the sepia diagram blank made in Step 2. The activity vector strips are best fixed to the diagram blank by means of rubber cement, and the activity interrelationships are delineated by arrows and schedule latitude symbols drawn directly on the diagram blank. Once completed, this master schedule can be reproduced for distribution. Again, slow printing speeds should be used for best results. Changes to the master schedule are easily made by pulling up the activity vector strips in question, erasing the arrows and latitude symbols with a soft eraser (light erasing will not affect the sepia diagram blank), recementing the usable activity vector strips into position, and finally drawing in arrows and latitude to complete the revision.

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* Trade name for E. I. du Pont de Nemours and Company's brand of polyester drafting film.

In certain cases, it may be desirable to use activity vector strips made from card stock during the initial scheduling phase, rather than paper or tracing paper. Once the vector strips are properly placed, they are removed one by one, and the activity vectors are drawn directly on the diagram blank along with the dependence arrows and schedule latitude symbols. Naturally, there is no need to draw formal activity vectors on the card stock strips; the identification code is all that is necessary. The resulting flow diagram can be reproduced by any standard method.

Appendix C

SOLUTION FOR THE EXAMPLE PROJECT ACCELERATED BY 10 DAYS

1. Start by compressing the critical path 3-13-15-17-18 (Figure 14) according to precedence.

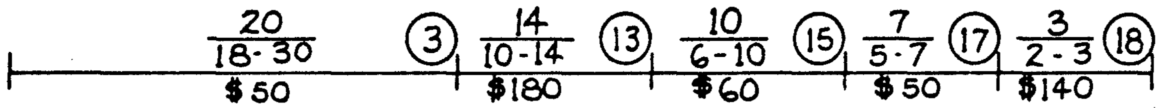

Figure C-1. Completion of Step 1.

2. Add path 1-5-7-11-17-18 (2 days schedule latitude). Two days of path compression can be taken by absorbing the latitude, thus 8 days of activity compression according to precedence are necessary. Activity 5 is compressed first by 4 days to its crash duration, then activity 17 is compressed by 2 days to its crash duration. Because of path coupling, the compression of activity 17 requires the extension of activity 3 (by 2 days). Activity 18 is compressed by 1 day to its crash duration, which requires the further extension of activity 3 by 1 day (duration of activity 3 is now 23 days). Activity 7 is compressed by 1 day to complete compression of the path. Activities 17 and 18 common to both paths have been compressed to their crash durations, thus no cost investigation across the node is possible, and the paths are coupled at lowest cost as they stand.

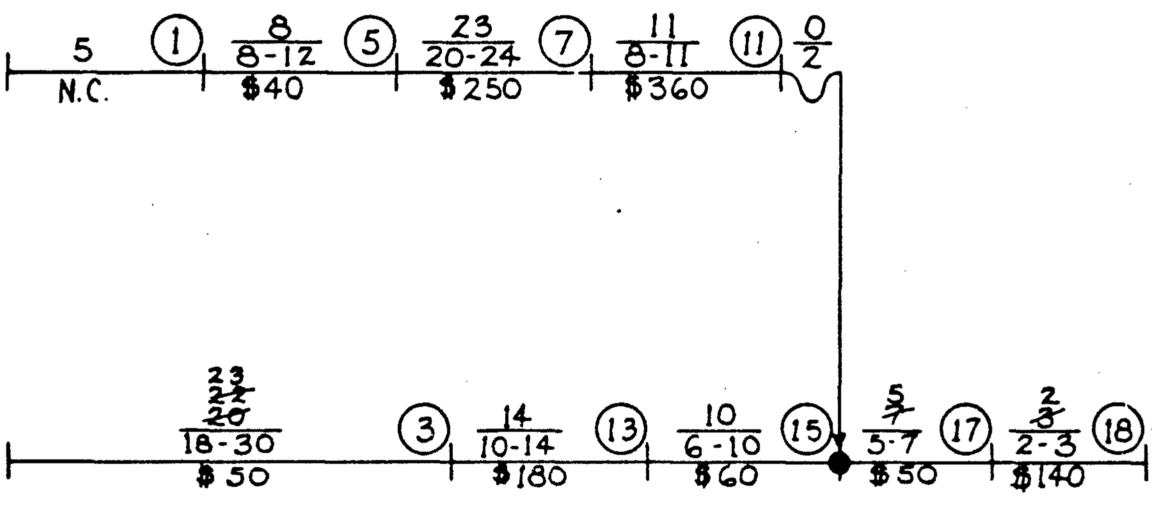

Figure C-2. Completion of Step 2.

3. Add path 2-10-14-16-18 (3 days schedule latitude). Three days of latitude can be absorbed, so 7 days of activity compression are necessary. Activity 18 has already been compressed by 1 day. According to precedence, activity 16 is compressed by 2 days to its crash duration, and activity 14 is compressed by the remaining 4 days required, to a duration of 17 days. The only common activity is activity 18 which has already been compressed to its crash duration, so again no cost investigation is possible — the paths are coupled at lowest cost.

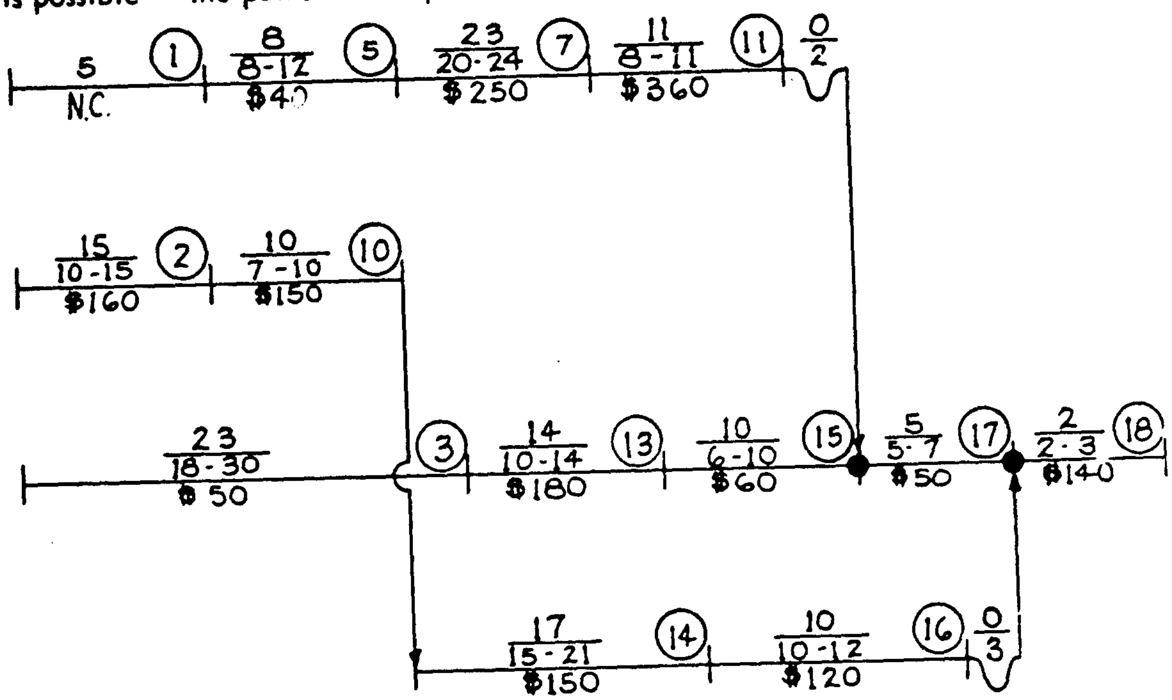

Figure C-3. Completion of Step 3.

4. Add path 2-10-13-15-17-18 (5 days schedule latitude). Five days of latitude can be absorbed; 5 days of activity compression are necessary. Activities 17 and 18 have already been compressed by a total of 3 days. According to precedence, activity 15 is compressed by the 2 days required, which demands the extension of activity 3 by 2 days. A cost investigation is necessary. The common activities which can be compressed are "2," "10," "13," and "15." Our attention is drawn first to activity 15 which has the lowest compression cost factor; however, the compression of this activity would require the extension of activity 18, and then possibly activity 17. Now, neither of these moves is advisable because it has already been shown that the compression of these activities to their crash durations results in the lowest path coupling cost. The alternative is to consider the compression of the common activity next in order of precedence — activity 10. The compression of

this activity would require the extension of activities 14 and 15; and the extension of activity 15, in turn, would require the compression of activity 3. A cost investigation shows that this move would cost $200, but would save $210. Consequently, activity 10 should be compressed. Once this activity has been compressed by 2 days, activity 15 has been extended as far as possible. Since activity 13 cannot be extended, no further duration changes can be made, and all the paths are coupled at lowest cost. The reader should remember that the compression of activity 10 resulted in the compression of activity 3 and the extension of activity 14 as well as the extension of activity 15.

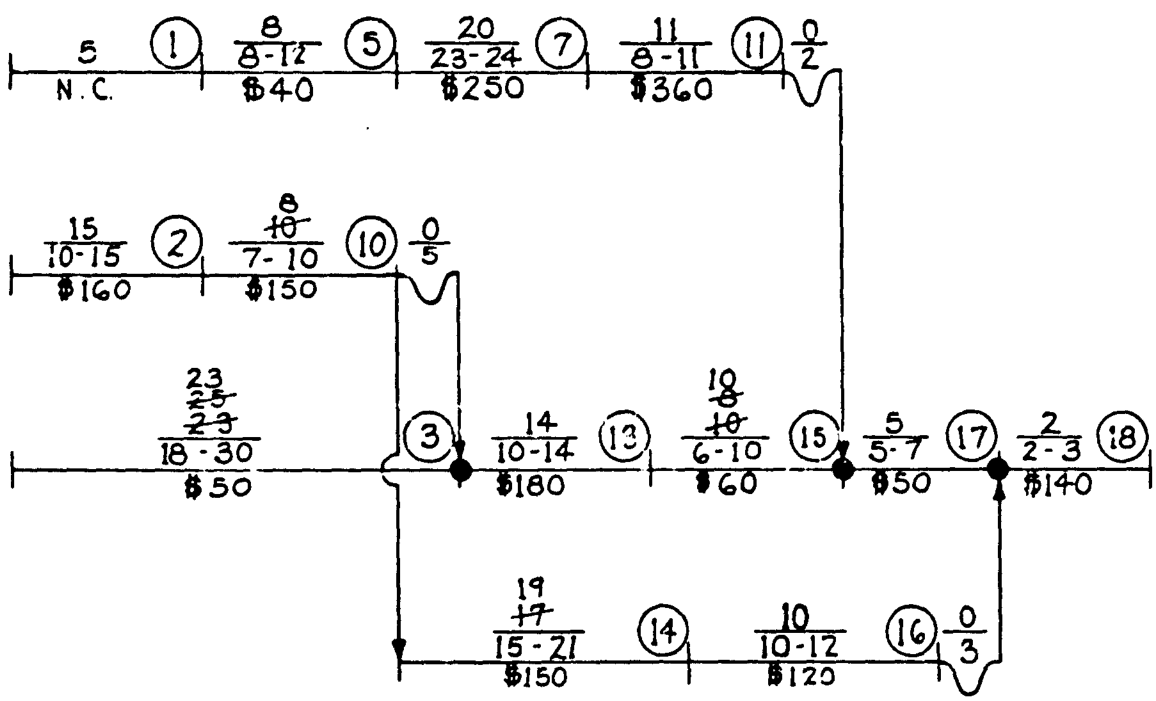

Figure C-4. Completion of Step 4.

Appendix D

EXTENSION OF THE OVERACCELERATED SCHEDULE AT LEAST COST

A direct approach to finding the least-cost project schedule for the crash duration is to develop an overaccelerated GPA flow diagram in which all activities are shown at their crash durations (Figure D-1). Inspection of this diagram shows that the critical path is 1-5-7-11-17-18, the same path resulting from the method discussed under the subject "Project Acceleration to the Crash Duration." Since activities on the critical path for the crash duration can be neither compressed nor extended, this path provides a reference frame within which noncritical activities can be extended to minimize project costs. Again, the cost-control diagram is very helpful for this purpose. Paths are added to the cost-control diagram in order of decreasing criticality, and cost precedence is the basis for determining which activities are to be extended. However, when activities are extended, cost precedence is based on the ranking of activities, in order of decreasing compression cost factors — just the opposite from compression precedence. When justification of a new path results in its becoming critical, a cost investigation across the node (when possible) determines whether or not a lowest cost for the coupled path has actually been achieved. To see how these procedures are employed, consider the example below. The critical path with all activities crashed has already been laid out on the cost-control diagram.

1. The first noncritical path to be considered is 2-10-14-16-18 (4 days schedule latitude). According to extension precedence, activity 2 should be extended to absorb the 4 days of latitude. This path is now critical. Even though activity 18 is common to both paths, it is shown at its crash duration; therefore no cost investigation can be made, and the paths are coupled at lowest cost.

2. Add path 1-6-10-14-16-18 (5 days total schedule latitude). Activity 6 is the only activity not already considered on the cost-control diagram. Since the other activities have not been extended into the latitude, activity 6 can be extended by 2 days up to its normal duration. Three days of latitude remain; consequently this path is noncritical, and no cost investigation across the node is necessary.

3. Add path 3-13-15-17-18 (7 days schedule latitude). Extension precedence indicates that activity 13 should be extended by 4 days to its normal duration, and that activity 15 should be extended into the remaining 3 days of latitude available. Activities 17 and 18 are common to other paths on the network, but since these activities are fully crashed no cost investigation is possible — the paths are coupled at lowest cost.

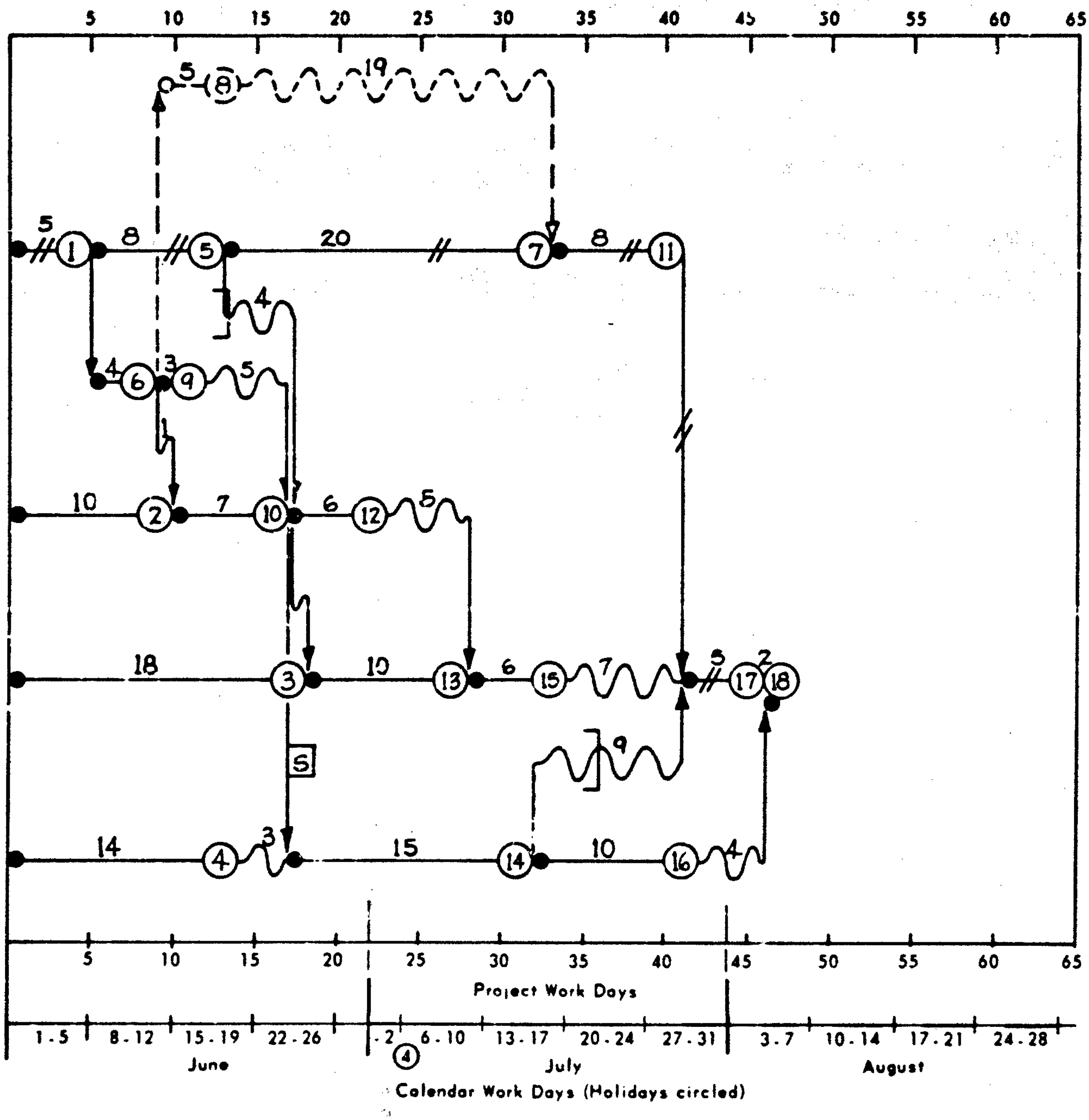

Figure D-1. GPA flow diagram schedule for the project overaccelerated at its crash duration.

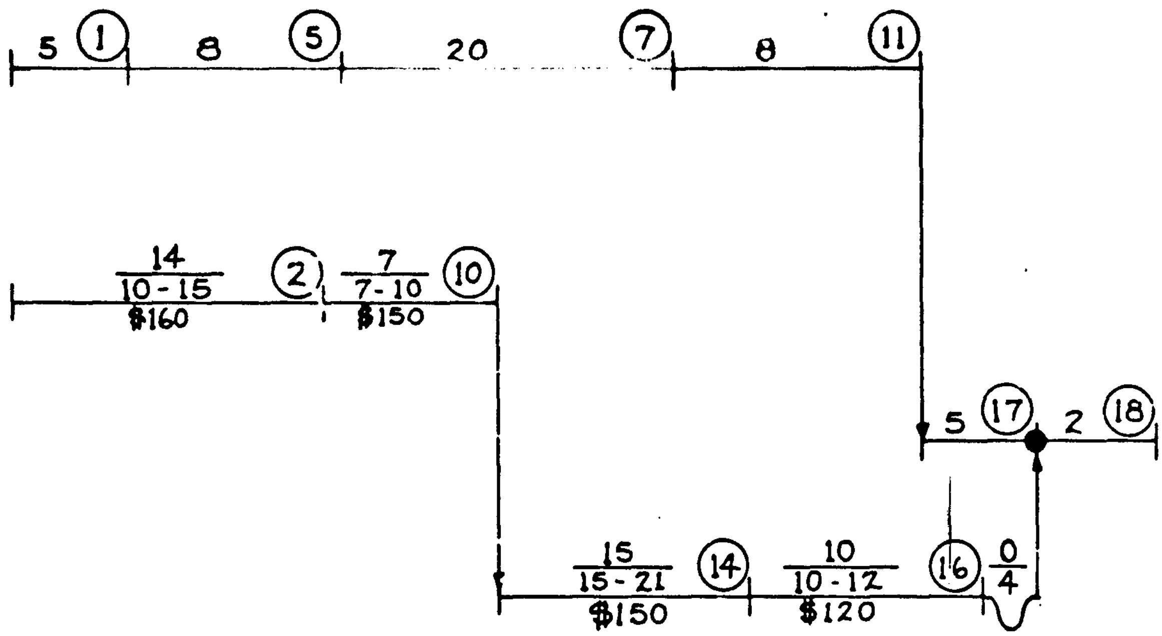

Figure D-2. Completion of Step 1.

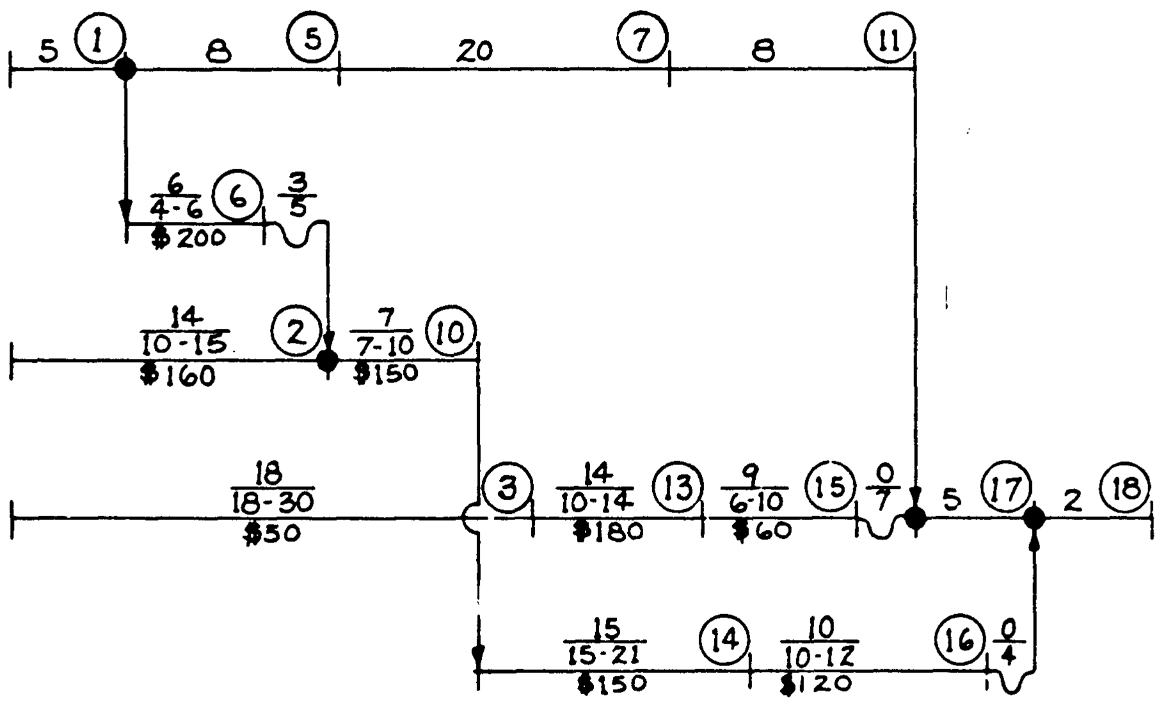

Figure D-3. Completion of Steps 2 and 3.

4. Add path 4-14-16-18 (7 days schedule latitude). Activity 4 is the only activity not already considered, and because none of the other activities have been extended into the latitude, activity 4 can be extended by 6 days to its normal duration. One day of latitude remains, and so no cost investigation is required.

5. Add path 2-10-13-15-17-18 (8 days schedule latitude). All activities have been considered, and activities 2, 13, and 15 have been extended. The total extension of these activities is 11 days, which exceeds the latitude available by 3 days. According to compression precedence, activity 15 should be compressed by the 3 days required. Since activity 15 also lies on path 3-13-15-17-18, which is critical, an activity on this path must be extended to compensate for the compression of activity 15; and only activity 3 can be extended (the duration of activity 3 becomes 21 days after extension). A cost investigation will determine if the paths can be coupled at a lower cost. Note that the only compressible activity common to both paths is activity 13. If this activity were compressed, activities 2 and 3 would be extended, and the extension of activity 2 would, in turn, require the compression of activity 14 and/or 16. But both of these activities are at their crash durations, therefore the proposed compression of activity 13 is impossible — the cost investigation cannot be carried out, and the duration assignments resulting from path justification must remain as they stand.

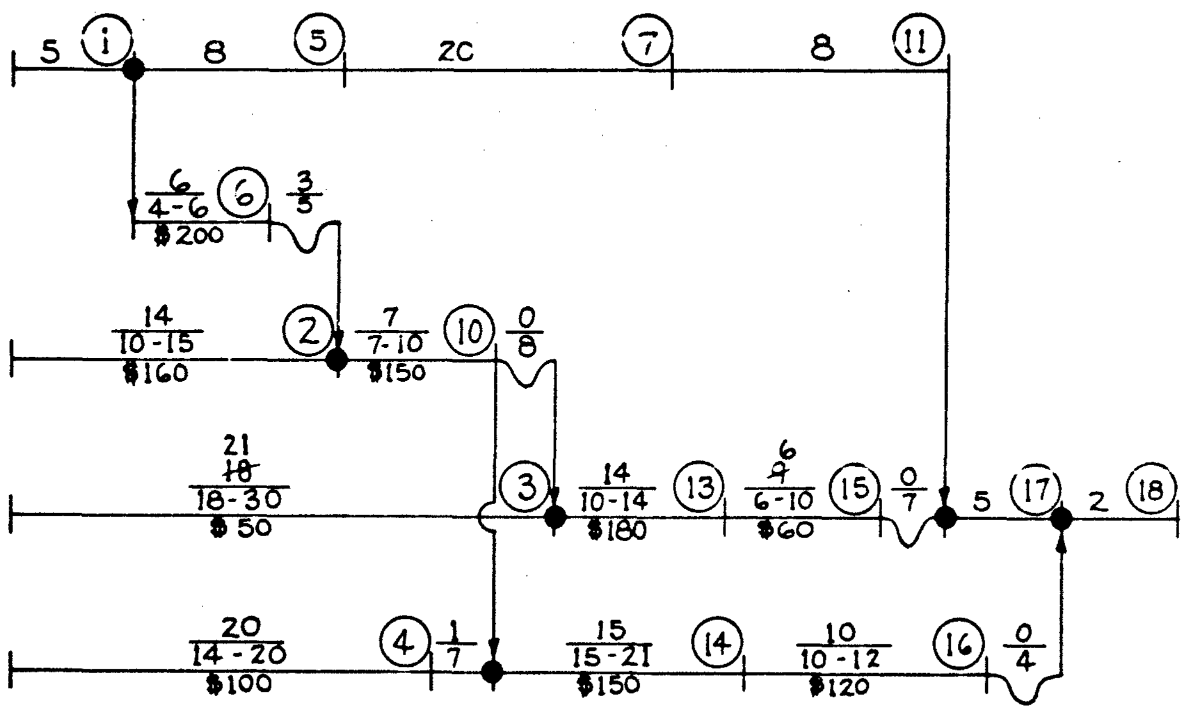

Figure D-4. Completion of Steps 4 and 5.

6. A comparison between the cost-control diagram and the overaccelerated flow diagram schedule shows that activities 8_g, 9, and 12 can be fitted into their respective paths at their normal durations without requiring any further path justification. Now all noncritical activities have been considered and selectively extended to minimize project costs. The activity durations derived on the cost-control diagram can be used to draw up a least-cost flow diagram schedule for the project's crash duration (Figure 21).

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